Extending ITL with Interleaved Programs for Interactive Verification

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joint work with
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TIME, Lübeck, 13.9.2011
General Setting:

- Specification of Software Systems with: Algebraic Specification, Z, Abstract State Machines (ASMs)
- Incremental Refinement of Designs: Algebraic, Data, ASM Refinement
- Verification of refinements: Tool support with KIV Interactive Verifier
Background: Proving Sequential Programs with KIV

KIV is an interactive theorem prover based on
- Structured algebraic specification of data types with higher-order logic
- Sequent calculus with proof trees
- wp-calculus for ASMs and Java
- Proof principle for sequential programs: symbolic execution (+ induction) [Burstall 74] (= incremental computation of strongest postconditions for instructions)
Concurrent systems: What Logic to use?

Define a general logic which

- allows proofs for arbitrary properties: safety, liveness, deadlock, fairness, refinement (trace inclusion)
- can handle systems specifications that use abstract data types
  → interactive proving approach
- provides modular support for various forms of concurrency:
  Programs with interleaving ("threading")
  Synchronous and asynchronous programs
  Harel- and UML-Statecharts
  (no encoding to transition systems)
Define a calculus where

- proving properties (e.g. contracts) for sequential programs should not be more difficult than using wp-calculus
- compositional reasoning (e.g. rely-guarantee) is supported, as otherwise concurrency generates too many cases

Content of my talk:

- One particular answer to choosing a logic and a calculus, based on ITL [Moszkowski 00].
- Some applications for interleaved programs.
Outline

- The Logic RGITL
  - Compositional interleaving
  - A semantics with system and environment steps
  - Integration with HOL
- Proof principles in RGITL
  - Symbolic Execution
  - Induction
  - Rely-Guarantee
- Application: Lock-Free Algorithms
  - Motivation
  - Simple Example: Treiber’s Stack
  - Linearizability and Lock-Freedom
- Experiences, Future Work
Why base the logic on ITL?

+ ITL directly offers termination/nontermination by using finite & infinite intervals
+ ITL is (easily) compatible with higher-order logic.
+ ITL offers the concept: programs $\subseteq$ formulas. The semantics of both is a set of intervals.
  - Some small extensions are needed:
    Is variable M in the program N := t?
    Recursive procedures
  - ITL does not offer a concept for interleaving.
Interleaving: Informal Semantics

Interleaved program \(\{ N := N^2; N := N^2 \} \parallel N := N + 1 \) started with \( N = 2 \):

- \( N = 17 \) → \( N = 25 \) → \( N = 81 \)
  - \( \uparrow \) \( \text{STEP} \)
  - \( N = 16, N := N + 1 \)
  - \( N := N^2, N = 5 \)
  - \( \downarrow \) \( \text{STEP} \)
  - \( N = 4, N := N^2 \parallel N := N + 1 \)
  - \( \downarrow \) \( \text{STEP} \)
  - \( N = 2, \{ N := N^2; N := N^2 \} \parallel N := N + 1 \)

Weak Fairness:

\{ \textbf{while} N \neq 0 \textbf{ do} N := N + 1 \} \parallel N := 0 \) terminates
Interleaving and Compositionality

A substitution rule is basic for a calculus to scale:

\[
\alpha \rightarrow A \quad \beta \rightarrow B \quad A \oplus B \rightarrow C
\]

\[\alpha \oplus \beta \rightarrow C\]

- holds in ITL for \( \oplus = \) sequential composition and other operators (similar to Hoare calculus)
- ideally, third premise should be trivial
- should hold for \( \oplus \) interleaving too!
Example for Noncompositional Interleaving in ITL

In classical ITL:

\{\textbf{while}^* N \neq 0 \textbf{do} N := 0\} \leftrightarrow \{\textbf{if}^* N \neq 0 \textbf{then} N := 0\} \quad (1)

(the star indicates, that the test does not take time)
Example for Noncompositional Interleaving in ITL

In classical ITL:

\[
\{ \text{while}^\ast N \neq 0 \ \text{do} \ N := 0 \} \leftrightarrow \{ \text{if}^\ast N \neq 0 \ \text{then} \ N := 0 \} \tag{1}
\]

(the star indicates, that the test does not take time)

Using the substitution rule:

\[
\{ \text{while}^\ast N \neq 0 \ \text{do} \ N := 0 \} \parallel \{ \text{while}^\ast N \neq 1 \ \text{do} \ N := 1 \} \tag{2}
\]

\[
\leftrightarrow \quad \{ \text{if}^\ast N \neq 0 \ \text{then} \ N := 0 \} \parallel \{ \text{while}^\ast N \neq 1 \ \text{do} \ N := 1 \} \tag{3}
\]

which is wrong:

(2) has nonterminating runs, which alternate between the loops
(3) terminates, since at some time \( N := 0 \) is executed
Example for Noncompositional Interleaving in ITL

In classical ITL:

\[ \{\text{while }^* N \neq 0 \text{ do } N := 0\} \leftrightarrow \{\text{if }^* N \neq 0 \text{ then } N := 0\} \]  \hfill (1)

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Using the substitution rule:

\[ \{\text{while }^* N \neq 0 \text{ do } N := 0\} \parallel \{\text{while }^* N \neq 1 \text{ do } N := 1\} \]  \hfill (2)

\[ \leftrightarrow \]

\[ \{\text{if }^* N \neq 0 \text{ then } N := 0\} \parallel \{\text{while }^* N \neq 1 \text{ do } N := 1\} \]  \hfill (3)

which is **wrong**:

(2) has nonterminating runs, which alternate between the loops

(3) terminates, since at some time \( N := 0 \) is executed

The problem is, that equivalence (1) ignores effects of the *environment* of the program
Basic idea: environment steps between program steps

- Semantics is based on Intervals $I =$ sequence of states $(I(0), I'(0), I(1), I'(1), \ldots )$
- state = valuation of variables
- $I$ has finite (termination!) or infinite length $\# I \in \mathbb{N} \cup \{\infty\}$
- $I$ alternates system steps $(I(0), I'(0)), (I(1), I'(1)), \ldots$ with environment steps $(I'(0), I(1)), (I'(1), I(2)), \ldots$
  (similar to reactive sequences [deRoever 01])
- Programs determine system steps only
- Primed and double primed (flexible) variables are needed: $X, X', X''$ denote the value of $X$ in $I(0), I'(0), I(1)$
  ($X = X' = X''$ in final states by convention)
Semantics of the Example in RGITL

The semantics of \textbf{while}^* N \neq 0 \textbf{do} N := 0 now are intervals where N has values \((n_i \neq 0)\):

- \((0)\)
- \((n_0, 0, 0)\) \(\text{ /* first env step does not change } N \text{ */}\)
- \((n_0, 0, n_1, 0, 0)\) \(\text{ /* env sets } N \text{ to } n_1 \text{ */}\)
- \((n_0, 0, n_1, 0, n_2, 0, 0)\)
- \(\ldots\)
- Nonterminating run \((n_0, 0, n_1, 0, n_2, 0, \ldots)\)
- \(\Rightarrow\) The two programs are not equivalent

But: equivalence is provable with environment assumption:
\[(\Box N' = N') \rightarrow \{\text{while}^* N \neq 0 \text{ do } N := 0\} \leftrightarrow \{\text{if}^* N \neq 0 \text{ then } N := 0\}\]
RGITL: Syntax

Extends simply types lambda-expressions with
- static (x) and flexible variables (X,X’,X’’)
- formulas (= expressions of type bool) with:
  - ◇, □, until, A, E /* all paths/exists path */,
  - ◯, • /* strong/weak next state */,
  - last /* termination */;
  - ; /* chop */;
  - * /* star */
  - ||, ||nf /* weak fair/nonfair interleaving */
  - p(T;Y) /* procedure call with input an in-out parameters */

TL and HOL operators can be freely mixed
RGITL: Semantics

- Expressions are evaluated over algebras (constructed as models of algebraic specs.) and an interval \( I = (I(0), I'(0), I(1), \ldots) \)
- If formula \( \varphi \) evaluates to true, write: \( I \models \varphi \)
- TL Operators have standard semantics:
  - \( (I(0), I'(0), I(1), I'(1), \ldots) \models \square \varphi \)
    iff for all \( n \leq \# I: (I(n), I'(n), I(n + 1), I'(n + 1), \ldots) \models \varphi \)
  - \( I \models A \varphi \) iff for all \( J \) with \( J(0) = I(0) \): \( J \models \varphi \)
  - \( I \models \text{last} \) iff \( I = (I(0)) \)
  - \( (I(0), I'(0), \ldots) \models \exists X. \varphi \)
    iff ex. \( (a_0, a'_0, \ldots) \) with \( (I(0)[X \leftarrow a_0], I'(0)[X \leftarrow a'_0], \ldots) \models \varphi \)
Programs in RGITL

- Programs $\alpha$ are formulas too: $I \models \alpha \iff$ the system steps in $I$ are possible steps of $\alpha$

- Programs: parallel assignments $X := T$, sequential (let, while, or, choose, rec. procedures) + $\alpha \parallel \beta$ (interleaving), await $C$ (block until $C$ holds)

- Programs $\alpha$ are placed in a frame assumption $[\alpha]_{X,Y}$ to indicate which variables are fixed in assignments (similar to TLA [Lamport 94], but no built-in stuttering)

- $[X := T]_{X,Y} \leftrightarrow X' = T \land Y' = Y \land \circ \text{last}$

- Typical goal: $\alpha \land E \rightarrow P$
  “Executing $\alpha$ in environment $E$ satisfies $P$”
Semantics of Interleaving

- Interleaving of two programs (or formulas) $\alpha$ and $\beta$ is defined compositionally, by interleaving individual intervals $\Rightarrow$ substitution rule is valid!
- Assume $I_1 \models \alpha$, $I_2 \models \beta$
- Interleaving gives all intervals $I$ which have
  - Interleaved system steps from $I_1$ and $I_2$ (fair)
  - The environment steps of $I_1$ ($I_2$) are the relevant alternating sequences of env. steps and system steps of $\beta$ ($\alpha$) in $I$

Formal def. in paper, including **blocked** steps (tricky):

$\text{await } \varphi \equiv \text{while}^* \neg \varphi \text{ do blocked}$
The Logic RGITL
  Compositional interleaving
  A semantics with system and environment steps
  Integration with HOL

Proof principles of RGITL
  Symbolic Execution
  Induction
  Rely-Guarantee

Application: Lock-Free Algorithms
  Motivation
  Simple Example: Treiber’s Stack
  Linearizability and Lock-Freedom

Experiences, Future Work
**Proof principle 1: Symbolic Execution**

- Symbolic execution = Step forwards through an interval
- Advantage: **no encoding of programs as transition systems with program counters** (as in Step, TLA or Model checking) ⇒ readable goals
- Symbolic execution is done in two phases: Unwinding and Stepping to the next state
Symbolic Execution: Unwinding (1)

- Splits formulas $\varphi$ with $X = \text{free}(\varphi)$ into formulas
  - $p(X, X', X'')$ describing the first step
  - $\circ \psi$ describing properties of the rest of the run

- Termination gives formulas of the form $q(X) \land \text{last}$

Examples:

\[
\square \varphi \equiv \varphi \land \bullet \square \varphi \\
\bullet \varphi \equiv \text{last} \lor \circ \varphi
\]

\[
[X := T; \alpha]_{X, Y} \equiv X' = T \land Y' = Y \land \circ [\alpha]_{X, Y}
\]

\[
[\text{let } X = T \text{ in } \alpha]_Y \equiv \exists X. (X = T \land [\alpha]_{X, Y} \land \square X' = X'')
\]

\[
[\text{choose } X \text{ with } \psi]_Y \equiv \exists X. (\psi \land [\alpha]_{X, Y} \land \square X' = X'')
\]

\[
[\text{in } \alpha \text{ ifnone } \beta]_Y \lor (\neg \exists X. \psi) \land [\beta]_Y
\]
Symbolic Execution: Unwinding (2)

To unwind interleaving and compounds unwind subprograms:

- If $\alpha \equiv p(X, X', X'') \land \circ \alpha'$ then

  \[
  \{\alpha; \beta\} \equiv p(X, X', X'') \land \circ \{\alpha'; \beta\}
  \]

  \[
  \{\alpha \parallel \beta\} \equiv \{\alpha <\parallel \beta\} \lor \{\alpha \parallel > \beta\}
  \]

  \[
  \{\alpha <\parallel \beta\} \equiv p(X, X', X'') \land \circ \{\alpha' <\parallel \beta\}
  \]

- If $\alpha \equiv q(X) \land \text{last}$ then

  \[
  \{\alpha; \beta\} \equiv q(X) \land \beta
  \]

  $\alpha <\parallel \beta \equiv q(X) \land \beta$
Symbolic Execution: Stepping

- Stepping removes the first step of interval: Instead of \((I(0), I'(0), I(1), I'(1), \ldots)\) consider \((I(1), I'(1), \ldots)\)

- Use new static variables \(x_0, x_1\) to store \(I(0)(X)\) and \(I'(0)(X)\) of the old first step in \(I(1)(x_0)\) and \(I(1)(x_1)\)

\[
\frac{p(x_0, x_1, X) \land \psi}{p(X', X'', X') \land \circ \psi} \quad \text{step} \quad \frac{q(x_0)}{q(X) \land \text{last}} \quad \text{last}
\]

- Effect: computation of the strongest postcondition of the first statement, weakened with environment assumption ⇒ sequential programs are executed as in wp-calculus

- Temporal properties result in (often non-temporal) additional goals for intermediate states
Proof principle 2: Induction

- Proofs use induction over well-founded orders
- Temporal induction reduced to well-founded induction by:
  \[ \lozenge \varphi \equiv \exists \ N. \ N = N'' + 1 \text{ until } \varphi \]
  “There is a number N of steps after which \( \varphi \) holds”
- Note that \( N = N'' + 1 \iff N'' = N - 1 \land N > 0 \)
- Proof of \( \Box \varphi \) by contradiction:
  Assume a number N of steps after which \( \neg \varphi \) holds
  Proof is then by well-founded induction over N
- Can be generalized to arbitrary safety properties
  (e.g. sequential programs without local variables)
Induction to prove Fairness

- Weak Fairness: In an interleaving $\alpha \parallel \beta$, program $\alpha$ eventually gets a chance to do a step (if not blocked)
- In TLA: separate formula talking about encoded steps with program counters \(\Rightarrow\) not an option of RGITL
- Alternative: General transformation of fair to unfair interleaved programs using counters [Apt, Olderog 91]
- In RGITL: Add an “$\alpha$ is scheduled flag” $B$:
  \[
  \{ B : \alpha \parallel \beta \} \iff \{ \alpha <\parallel \beta \} \lor (\neg B \land \{ B : \alpha \parallel> \beta \})
  \]
- New Axiom: $\{ \alpha \parallel \beta \} \iff \exists B. \diamond B \land \{ B : \alpha \parallel \beta \}$
- $\diamond B$ allows induction!
- Unfair interleaving satisfies almost the same axiom:
  \[
  \alpha \parallel_{nf} \beta \equiv (\exists B. \diamond B \land \{ B : \alpha \parallel_{nf} \beta \})
  \lor (\beta \land \Box (\neg \text{blocked}) \land E \exists X. \alpha)
  \]
- $E \exists X. \alpha$: “there is at least one run of $\alpha$” ($X = \text{free}(\alpha)$)
Proof principle 3: Compositional Reasoning

- Substitution principle allows to abstract each program in an interleaving to a property
- In particular: Rely/Guarantee rules are expressible
- Guarantee = Predicate for steps of a process $G(X,X')$
- Rely = Predicate on environment steps $R(X',X'')$
- Program $\alpha$ satisfies R/G, iff:

$$\alpha \subseteq R \subseteq G$$

As a TL formula: $R \rightarrow G \equiv \neg (R \text{ until } (\neg G))$

*(not a special operator as in TLA [Lamport 94]!)*
Proof principle 3: Compositional Reasoning

- Basic principle:
  - Prove $R_i/G_i$ for interleaved programs $\alpha_i$ ($i = 1,2$)
  - Prove $G_i \rightarrow R_j$ for $i \neq j$, $R_i$ transitive
  - Then: $\alpha_1 \parallel \alpha_2$ satisfies $\square G_1 \lor G_2$

- Provable by using the substitution principle, with
  $A \equiv R_1 \rightarrow G_1$, $B \equiv R_2 \rightarrow G_2$, $C \equiv (X' = X'') \rightarrow G_1 \lor G_2$

\[
\begin{align*}
\alpha_1 \rightarrow A & \quad \alpha_2 \rightarrow B \\
A \parallel B \rightarrow C & \quad \alpha_1 \parallel \alpha_2 \rightarrow C
\end{align*}
\]

- First two premises = Assumptions for the two programs
- Third premise provable by induction, using
  $R \rightarrow G \leftrightarrow \forall B. \Diamond B \rightarrow (R \land \neg B) \rightarrow G$
Rely-Guarantee Theorem

<table>
<thead>
<tr>
<th>Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( pre \land COp_1 \rightarrow R_1 \xrightarrow{+} (G_1 \land (last \rightarrow post_1)) )</td>
</tr>
<tr>
<td>(2) ( pre \land COp_2 \rightarrow R_2 \xrightarrow{+} (G_2 \land (last \rightarrow post_2)) )</td>
</tr>
<tr>
<td>(3) ( G_1 \lor R \rightarrow R_2, G_2 \lor R \rightarrow R_1, G_1 \lor G_2 \rightarrow G )</td>
</tr>
<tr>
<td>(4) reflexive((G_1, G_2)), transitive((R_1, R_2))</td>
</tr>
<tr>
<td>(5) ( pre \land (R_1 \lor R_2) \rightarrow pre )</td>
</tr>
</tbody>
</table>

then \( pre \land COp_1 \parallel COp_2 \rightarrow R \xrightarrow{+} (G \land (last \rightarrow post_1 \land post_2)) \)

- similar to [Xu, deRoever 97] (except cond. (5))
- Their notation for (1): \( COp_1 \textit{ sat} (pre, rely_1, guar_1, post_1) \)
- Deadlock freedom provable too (using \textit{blocked} \rightarrow \textit{wait})
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  - Linearizability and Lock-Freedom
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Motivation

- Multi-core processors getting more and more common ⇒ Concurrent algorithms more important than ever
- Usually, concurrency is implemented using locks (semaphores, synchronize in Java etc.)
- Lock-free algorithms (also called nonblocking) are an interesting class of algorithms that does not use locks
- Instead they use **CAS instructions** (x86, Sparc, Itanium) or LL/SC (Alpha, PowerPC)
Example: Treiber’s Stack

- Defined in [Treiber 86]
- Implementation of a global stack
- Abstract view: Operations APush and APop
- Implementation with algorithms CPush and CPop
- Representation of stack as a linked list.
CPush(v : Data; top : Pointer) {
    n := new(Node);
    n.val := v;
    success := false;
    while success = false do {
        tmp := top;
        /* other process .. */
        /* .. may change top! */
        n.next := tmp;
        CAS(tmp, n, top)
    }
    CAS(tmp, n, top; success) { /* atomic! */
        if* top = tmp
            then { top := n, success := true; }
        else success := false; }
}
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    while success = false do {
        tmp := top;
        /* other process .. */
        n.next := tmp;
        CAS(tmp, n, top)
    }
}

CAS(tmp, n, top; success) { /* atomic ! */
    if* top = tmp
    then { ; }
    else success := false; }

Diagram:

- n
- v
- tmp
- top
- Node
- Data
Lock-Free Algorithms and their Use

- Principle of lock-free algorithms:
  - read old data structure
  - prepare modified version
  - update with CAS. Retry on failure

- Treiber’s Stack is one of the simplest algorithms (inefficient for high loads; better: [Hendler et al. 04])

- Lock-Free Algorithms exist for many data structures:
  - Queues [Michael, Scott 96], Hashtables [Michael 02], [Gao et al. 05], Linked Lists [Harris 01], [Heller 05]

- Used for: process queues, indexes of data bases and Web Servers, real-time 3D games, garbage collection

- Java library supports CAS; f implements lock-free data structures
Advantages of using Locks:

- Well understood, uniform principle
  ⇒ easier to verify than lock-free algorithms
  (essentially: verify sequential algorithm)
- Automatic checks for correct use of locks available
- Simple lock-free algorithms are inefficient at high loads: they waste processor time trying over and over

Disadvantages of using Locks:

- Lock is a bottleneck (pessimistic strategy)
- Deadlocks and priority inversion possible
- What happens when the locking process crashes?
Safety: Linearizability

- Defined in [Herlihy & Wing 90]
- Scenario: Several processes \((p,q,r)\), all running algorithm COp in parallel (e.g. CPush \(\lor\) CPop)
- Informal definition: Parallel run must be equivalent to a sequential run of AOp (APush \(\lor\) APop)

```
[] APush_p(a) [a] APop_q(a) [] APush_r(b) [b]

inv_p(CPush,a) inv_q(CPop) ret_q(CPop,a) ret_p(CPush)

inv_r(CPush,b) ret_r(CPush)
```

Timeline
Decomposition of Linearizability

**Theorem (Bäumler et al. 09)**

If for all \( 1 \leq p, q \leq n, p \neq q \):

1. \( COp_p \rightarrow R_p \xrightarrow{+} G_p \)
2. \( G_p \rightarrow R_q, \text{ reflexive}(G_p), \text{ transitive}(R_p), R \rightarrow R_p \)
3. \( COp_p(CS) \land \square (R_p \land Abs(CS) = AS \land Abs(CS') = AS') \rightarrow \text{skip}^*; AOOp_p(AS); \text{skip}^* \)

then \( COp_1^* \parallel \ldots \parallel COp_n^* \land \square R \rightarrow AOOp_1^* \parallel \ldots \parallel AOOp_n^* \parallel \text{skip}^* \)

- \( COp_p \) is a concrete algorithm (procedure) that implements an atomic operation \( AOOp_p \)
- \( R \) is the global environment assumption
- Linearizability expressed as special case of refinement
- Most linearizable algorithms allow reduction to two representative processes \( \Rightarrow \) reduction proved
Liveness: Lock-Freedom

For Treiber’s Stack:

- CPush may have to retry over and over
  ⇒ one single process might be starved
- Every time a retry is necessary, another CPush/CPop must have succeeded and terminated
- This is true, even if the scheduling is unfair, or when a process crashes

Treiber’s stack satisfies property of Lock-Freedom:

As long as some operations are running, one of them will terminate
Decomposition of Lock-Freedom

**Theorem (Tofan et al. 10)**

If for all $0 \leq p, q, p \neq q$:

1. $COp_p \rightarrow R_p \rightarrow G_p$
2. $G_p \rightarrow R_q$, reflexive($G_p$), transitive($R_p$), $R \rightarrow R_p$
3. reflexive($U$), transitive($U$), $R \rightarrow R_p \land U$
4. $COP_p(CS) \land \Box R_p$
   
   \[ \rightarrow \Box (\neg U(CS, CS') \lor (\Box U(CS', CS''))) \rightarrow \Diamond \text{last} \]

then $COP_0^* \| \ldots \| COP_n^* \land \Box R \rightarrow \Box \text{progress}$

where progress = “some operation active $\rightarrow$ some operation terminates”

- Predicate $U$ (“unchanged”) describes conditions under which $COP_p(CS)$ terminates in environment $R_p$.
- At any time, $COP_p$ eventually terminates ($\Diamond \text{last}$), if:
  - It updates the shared state itself $\neg U(CS, CS')$, or
  - It encounters no interference $\Box U(CS', CS'')$
- Theorem holds for weak fair and nonfair interleaving
Symbolic execution is natural to verify even concurrent programs:
- rest of the program directly visible
- feels much like debugging

Main new difficulty for proofs is to determine the correct Relys and Guarantees (similar to invariants) in advance ⇒ Add techniques to automatically infer them

We’ve done some significant case studies already:
- Hazard pointers for lock-free algorithms [⇒ tomorrow]
- Medical protocols with synchronous parallel hierarchical plans [Protocure 06]

Calculus is not yet as easy to use or automated as the wp-calculus for sequential programs (takes time and experience)
Some Open Issues

- Guarantees often hold in a certain section of the code: currently boolean variables must be added manually ⇒ labels would be helpful, but are incompatible with chop: $\alpha;\{L : \beta\}$: final state of $\alpha$ and first of $\beta$ disagree on $L$
- express general refinement modulo stuttering
- Prove general commuting diagrams for forward and backward simulation (bounded nondeterminism!)
- Completeness in general is open (complete fragments of ITL and RG)
Proving Lock-Free Algorithms

- Calculus is adequate to show correctness of proof obligations (POs) as well as proving instances of the POs for case studies.

- Automation is not as high as in related work:
  Automatic checking of linearizability for short operations sequences with model checking [Alur10]
  Automatic proofs for some algorithms using RGSep [Vafeiadis01]

- Nevertheless, the algorithms we check are already more difficult than those that have been proved automatically.
Current Work on Lock-Free Algorithms

- Support for Heap modularity is often beneficial
  ⇒ develop library with a lightweight embedding of separation
- Open issue: good frame rule for temporal logic?
- Generalize proof obligations (POs) for linearizability
  (POs shown require lin. points within executing thread
  ⇒ complete POs for arbitrary lin. points

Major Challenge:
Interleaving assumes sequentially consistent memory, but:
Processors use weak memory models
(and Java's much debated memory model is even weaker)