From Priced Timed Automata to Energy Games

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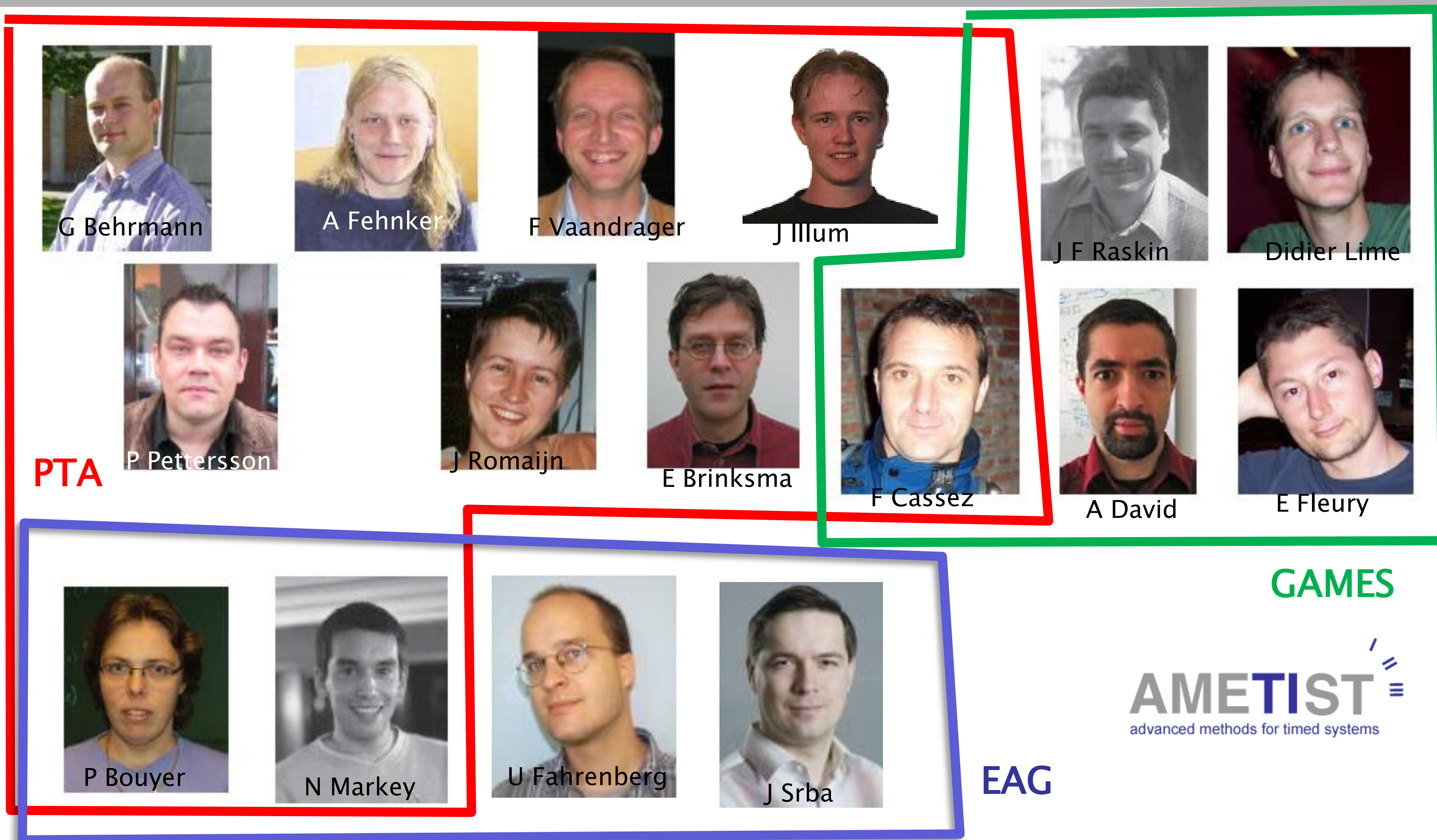
Collaborators



Overview

- Timed Automata
- Priced Timed Automata
  - Optimal Reachability
  - Optimal Infinite Runs
  - Priced Timed Games
- Energy Automata & Games
  - 1 Clock & 1 Cost
  - 1½ Clocks & 1 Cost
  - Multiple Clocks & Cost
  - Multiple Clocks & 1 Cost
  - Multiple Clocks & 0 Cost
-

Overview

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- Energy Automata & Games
  - 1 Clock & 1 Cost
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  - Multiple Clocks & Cost
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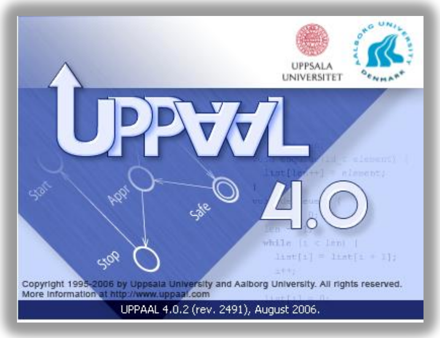
  - Multiple Clocks & 0 Cost
- Tool Support= Statistical Model Checking!



CLASSIC

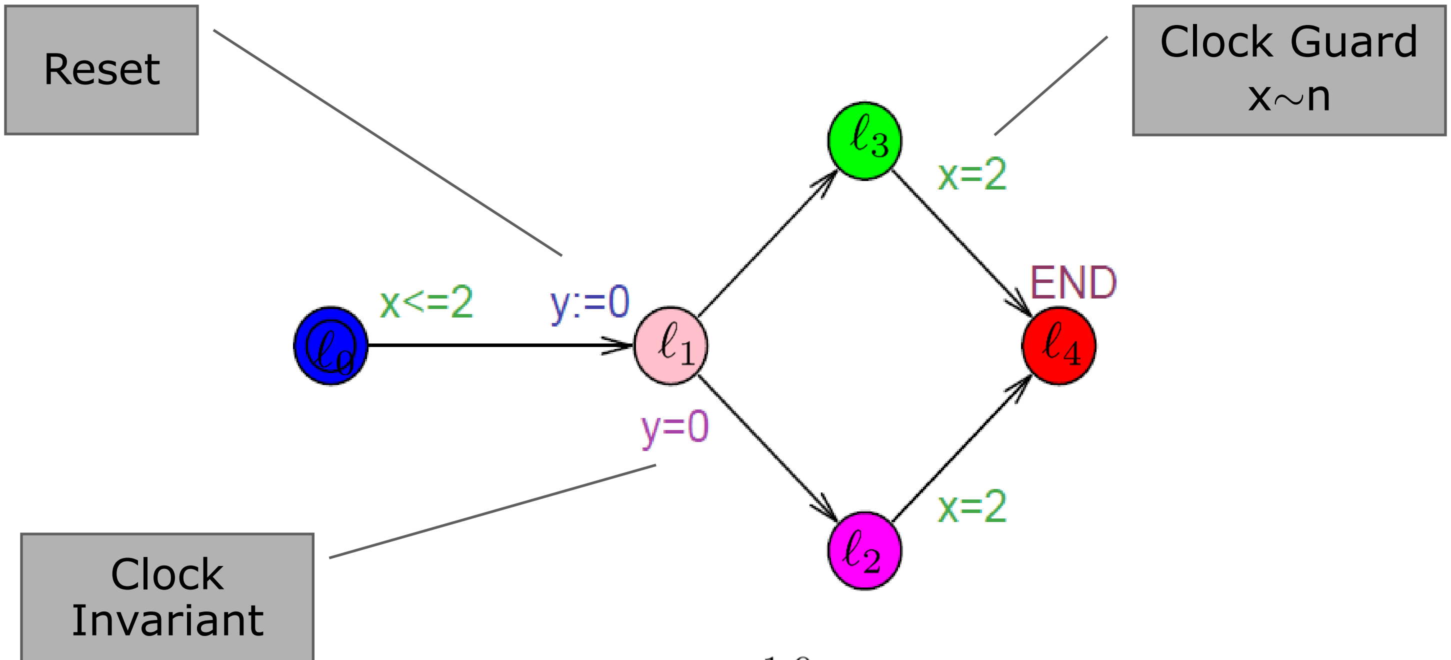
CORA

TIGA

SMC

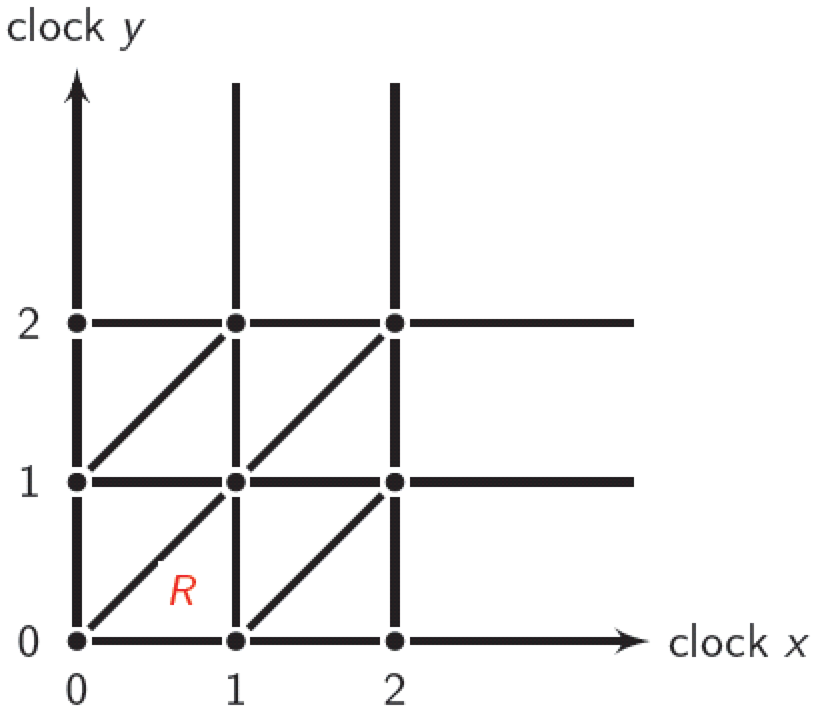
Timed Automata

Timed Automata [Alur & Dill'89]



$$
\begin{aligned}
(\ell_0, x=0, y=0) &\xrightarrow{1.9} (\ell_0, x=1.9, y=1.9)\\
&\longrightarrow (\ell_1, x=1.9, y=0)\\
&\longrightarrow (\ell_2, x=1.9, y=0)\\
&\xrightarrow{0.1} (\ell_2, x=2.0, y=0.1)\\
&\longrightarrow (\ell_4, x=2.0, y=0.1)
\end{aligned}
$$

The Region Abstraction



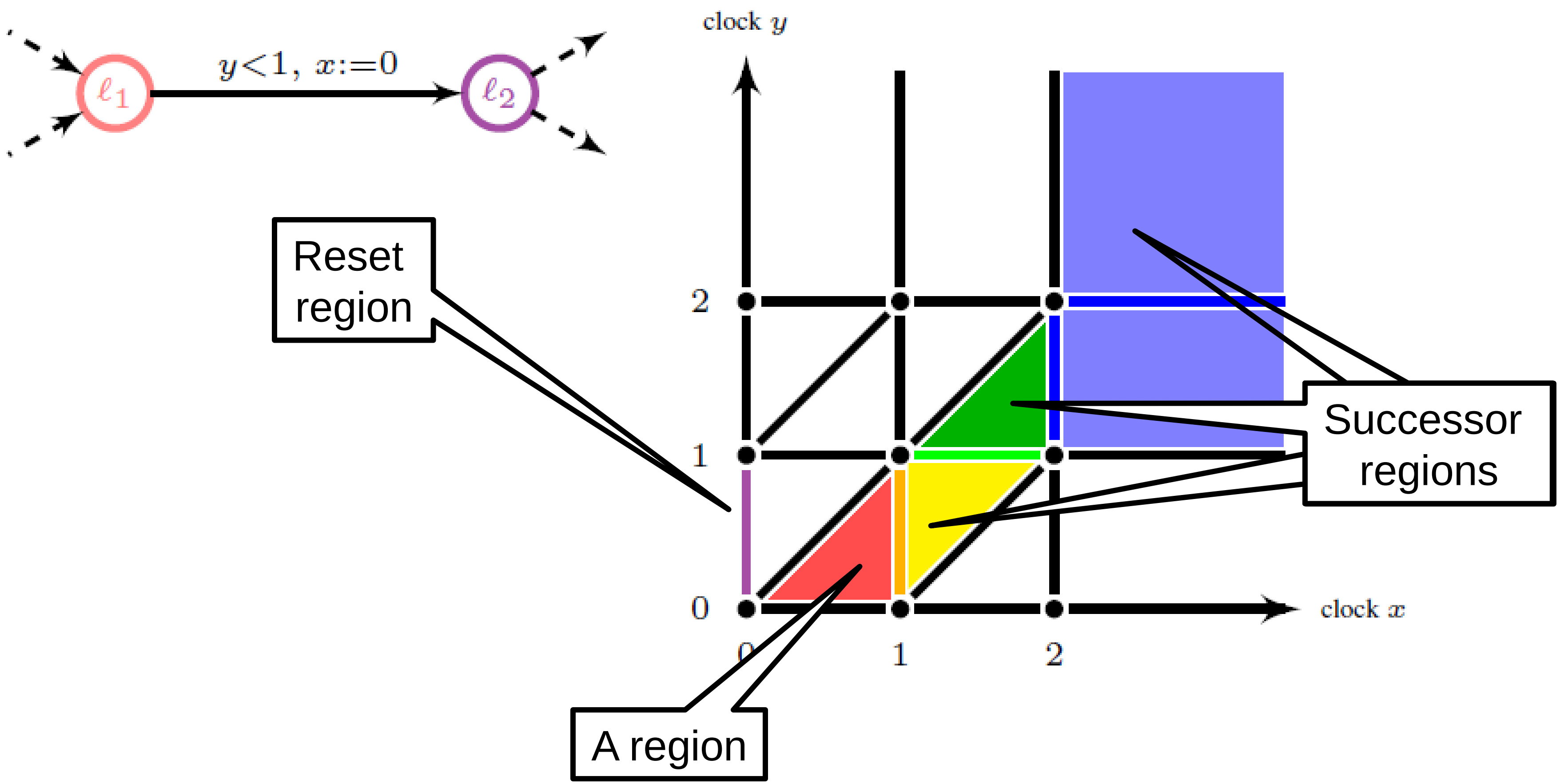
region R defined by:

$$\begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ y < x \end{cases}$$

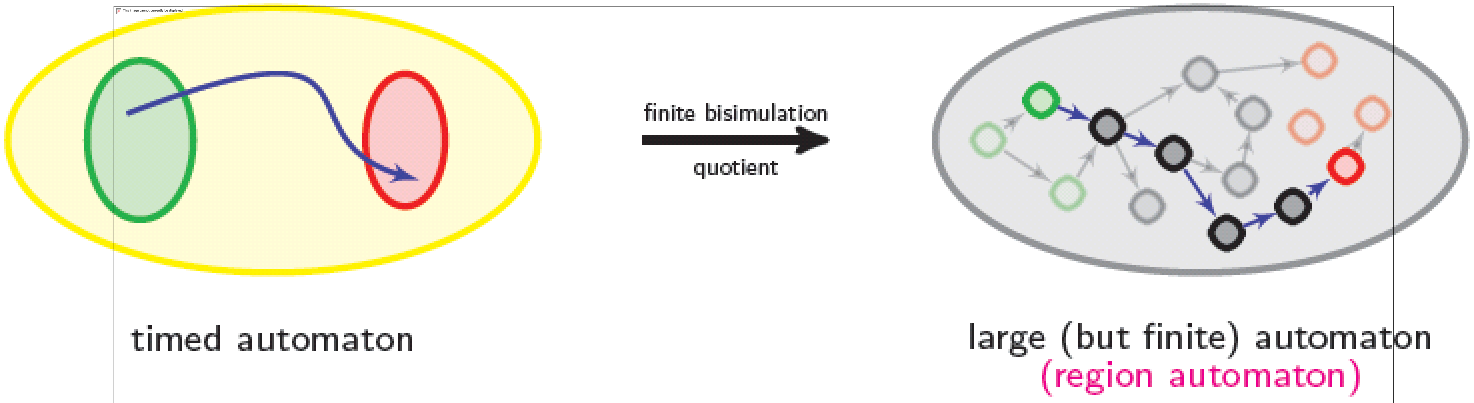
- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

$\leadsto$ an equivalence of finite index
a time-abstract bisimulation

Regions – From Infinite to Finite



Region Automaton



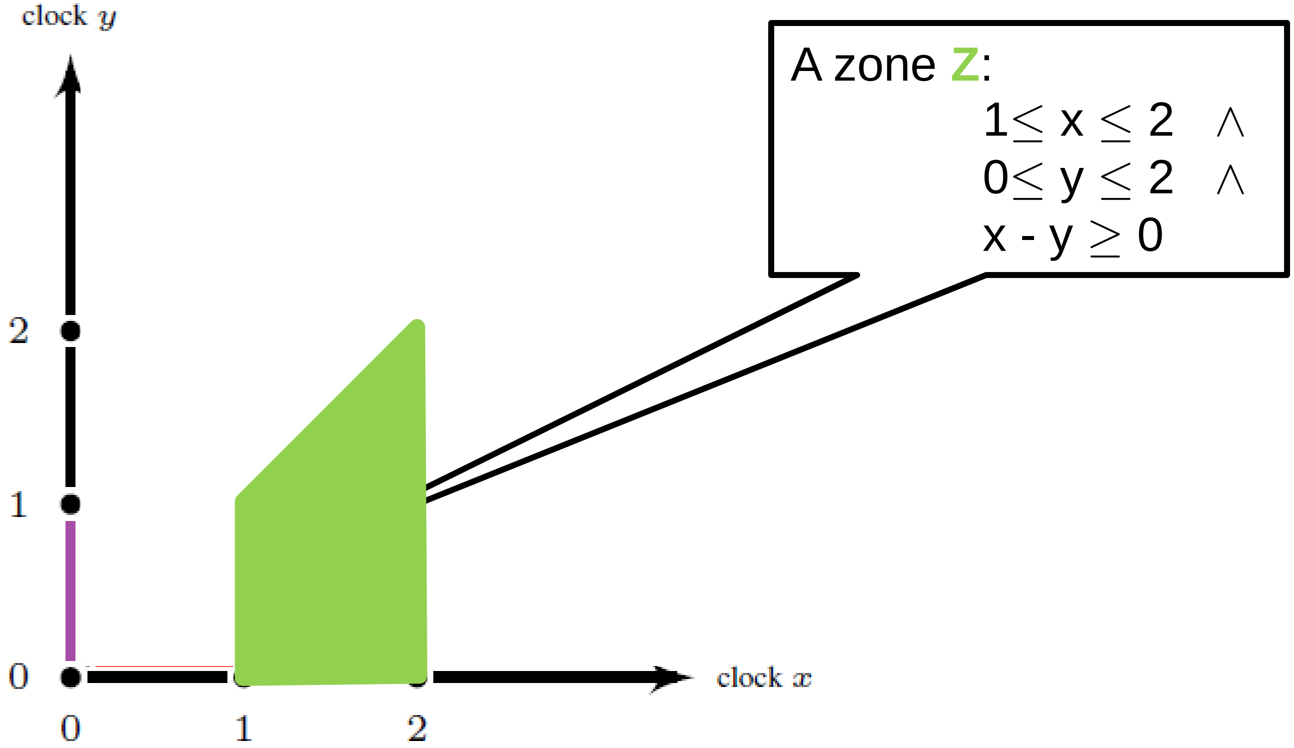
THM [AD90]
Reachability is decidable (and PSPACE-complete) for timed automata

THM [CY90]
Time-optimal reachability is decidable (and PSPACE-complete) for timed automata

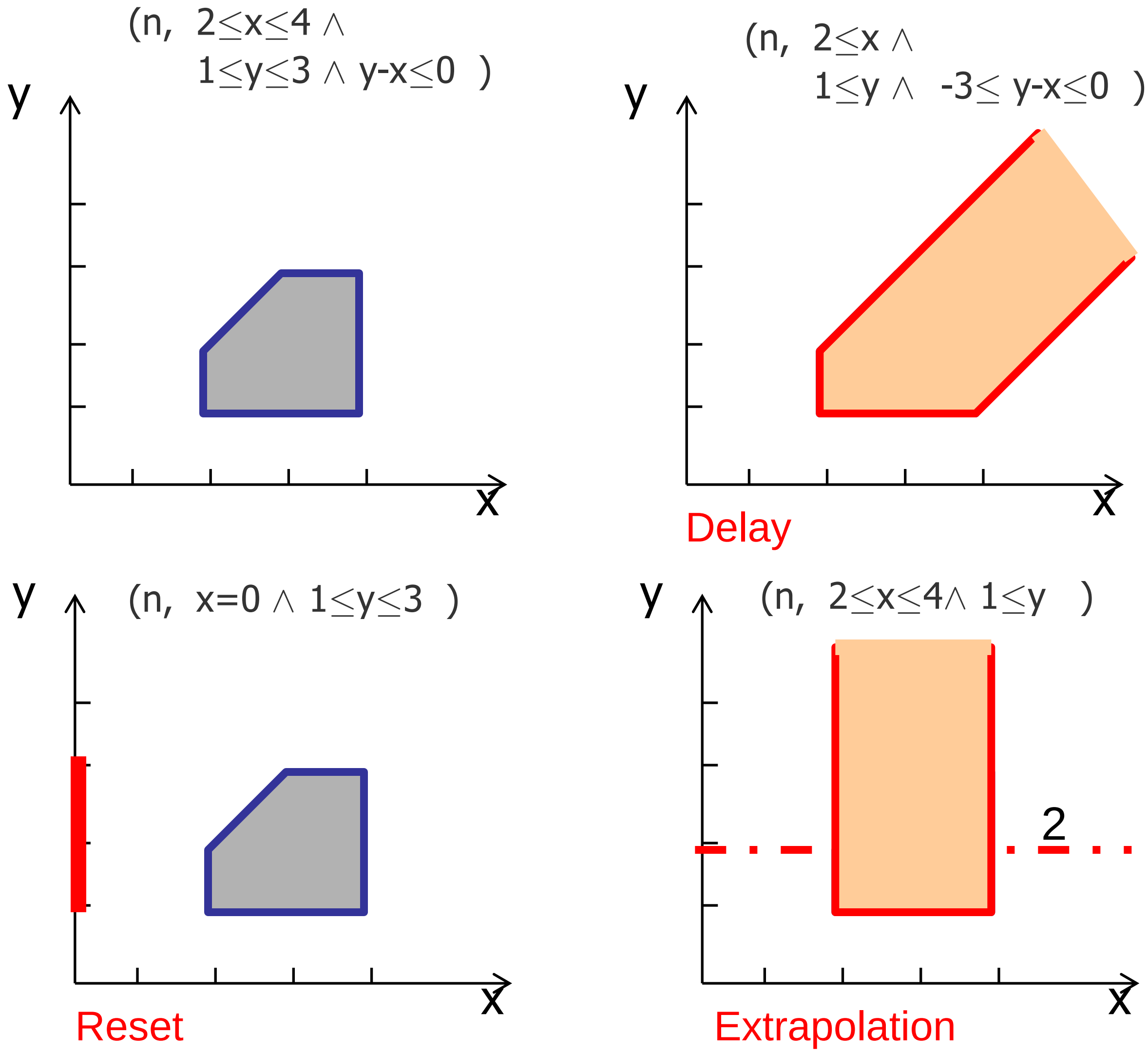
LARGE: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is

$$\prod_{x \in X} (2M_x + 2) \cdot |X!| \cdot 2^{|X|}$$

Zones – From Finite to Efficiency

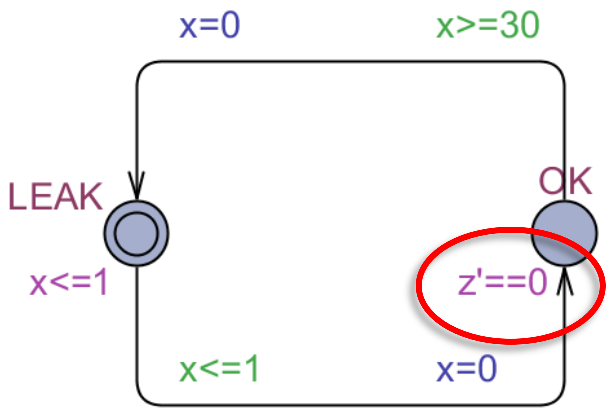


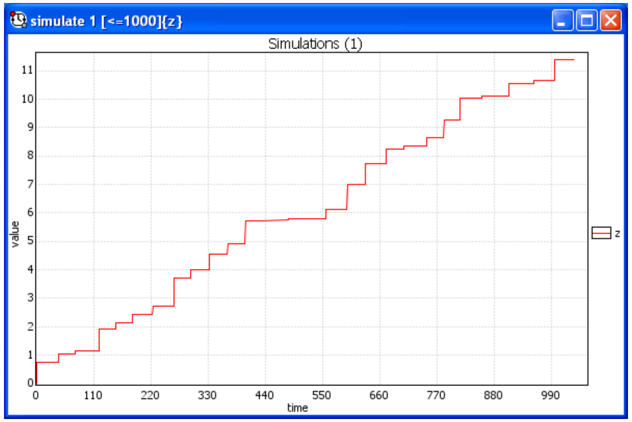
Zones – Operations



Beyond Time

- **Hybrid Automata**: timed automata augmented with variables whose derivative are not constant.



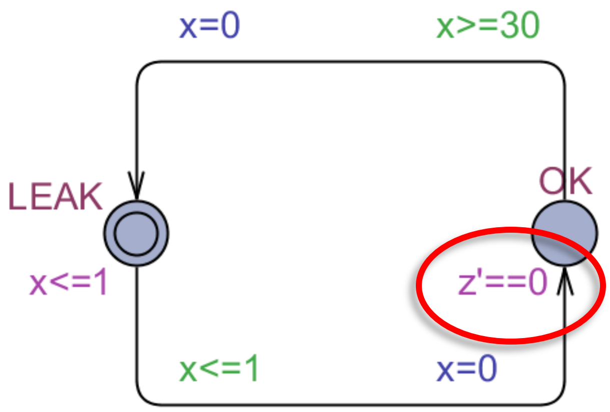


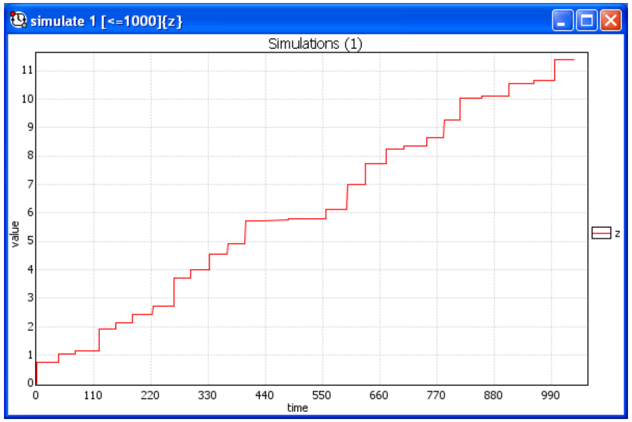
- E.g: leaking gas burner, water-level monitor.
- **THEOREM**: reachability **undecidable** (even for a single stop-watch)

Refs: [1] Henzinger, Kopke, Puri, Varaiya. *What's Decidable about Hybrid Automata?* (1995).

Beyond Time

- **Hybrid Automata**: timed automata augmented with variables whose derivative is not constant.



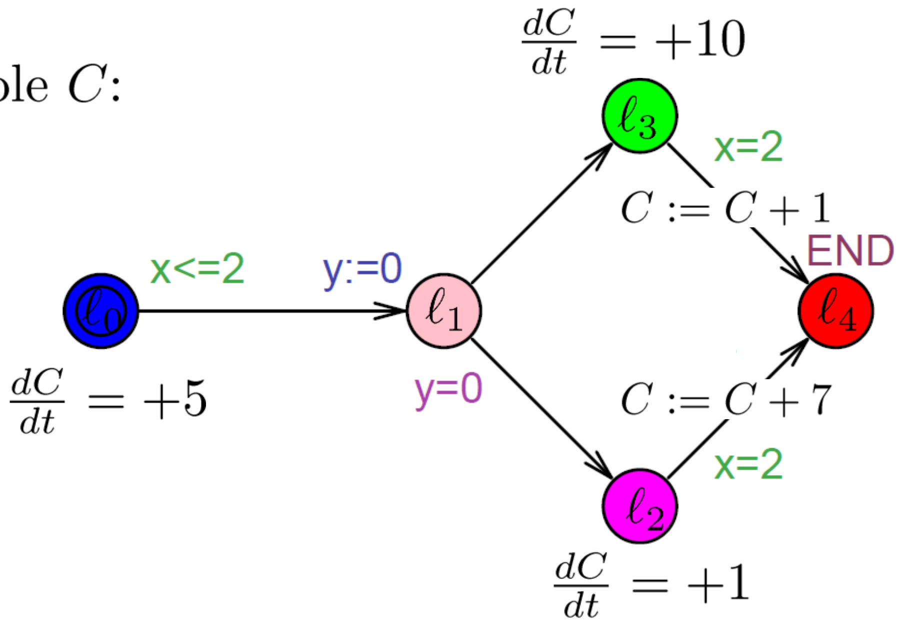


- **Time automata with observers**: as hybrid automata, but behaviour only depends on clocks

Priced Timed Automata

A simple example

Observer variable C:



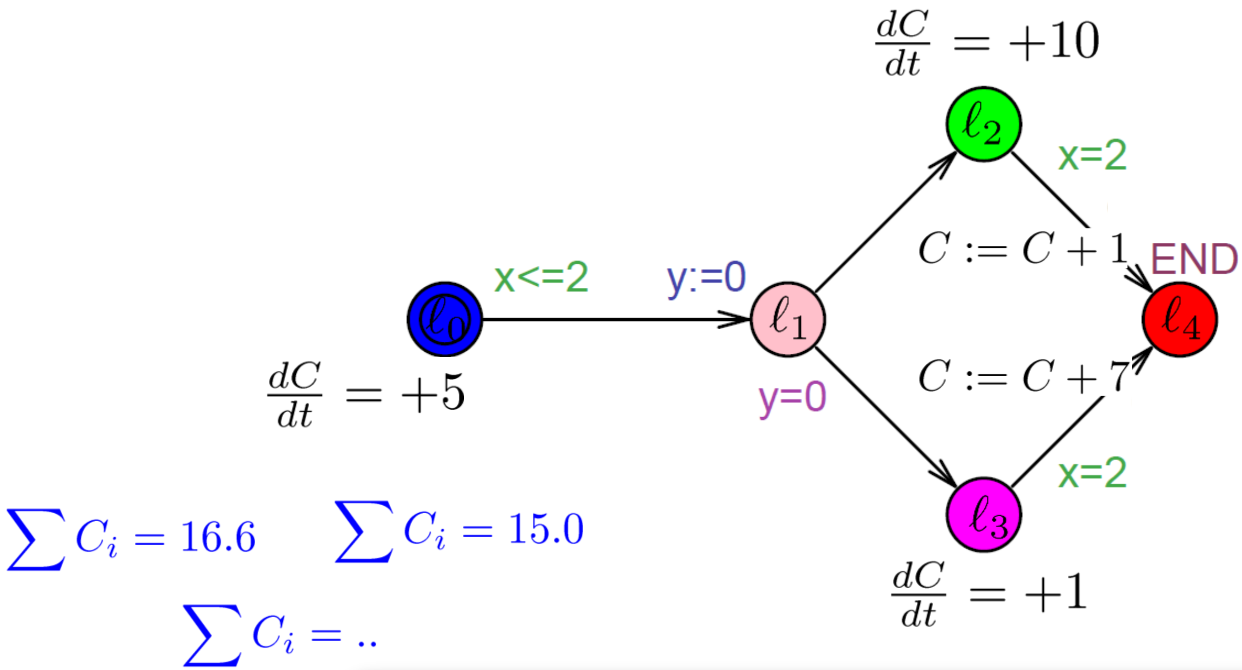
$$(\ell_0,[0,0]) \xrightarrow{1.9}_{9.5} (\ell_0,[1.9,1.9]) \rightarrow_{0} (\ell_1,[1.9,0]) \rightarrow_{0} (\ell_2,[1.9,0]) \xrightarrow{0.1}_{0.1} (\ell_2,[2,0.1]) \rightarrow_{7} (\ell_4,[2,0.1])$$

$$\sum C_i = 16.6$$

$$(\ell_0,[0,0]) \xrightarrow{1.2}_{6.0} (\ell_0,[1.2,1.2]) \rightarrow_{0} (\ell_1,[1.2,0]) \rightarrow_{0} (\ell_3,[1.2,0]) \xrightarrow{0.8}_{8.0} (\ell_3,[2,0.8]) \rightarrow_{1} (\ell_4,[2,0.8])$$

$$\sum C_i = 15.0$$

A simple example

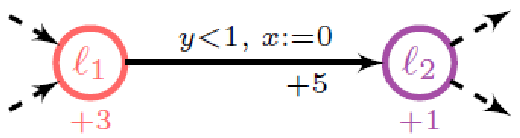


Q: What is cheapest cost for reaching ℓ_4 ?

$$\inf_{0 \leq t \leq 2} \min\{5t + 10(2-t) + 1, 5t + (2-t) + \mathbf{7}\} = 9$$

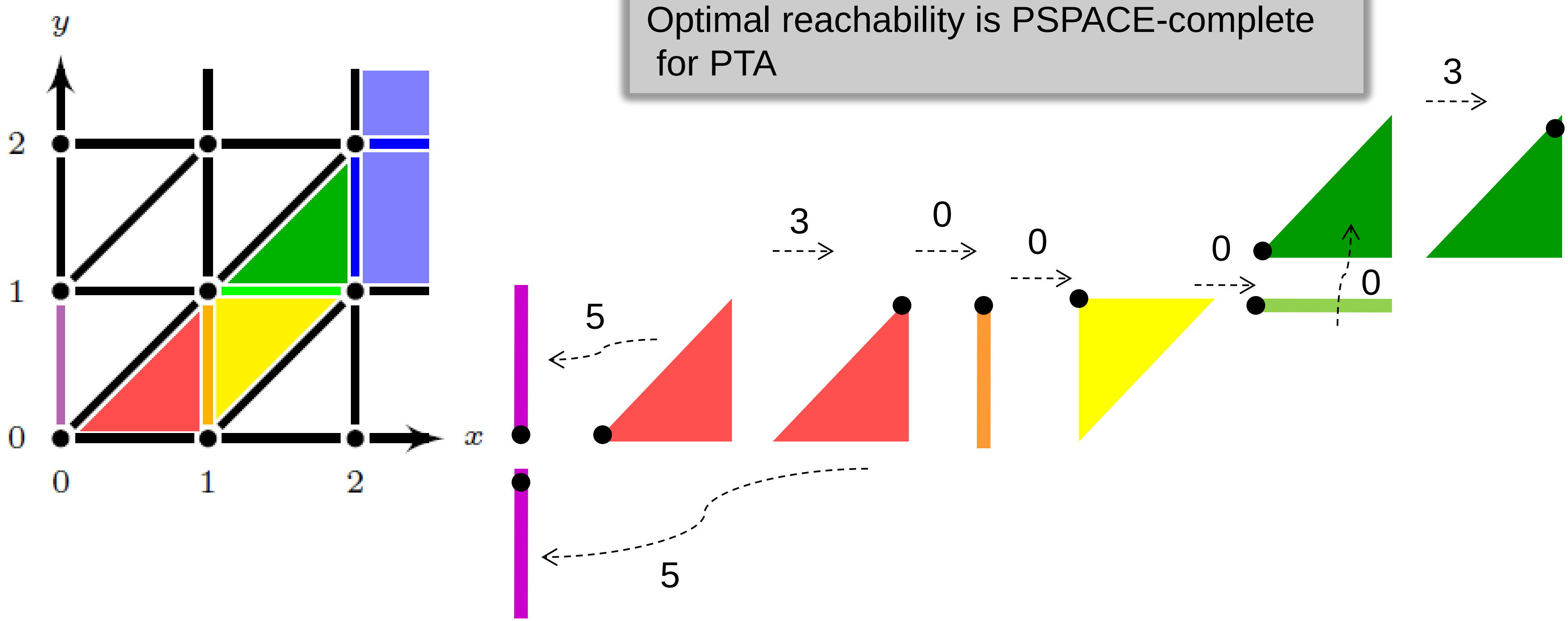
➜ **strategy:** leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

Corner Point Regions



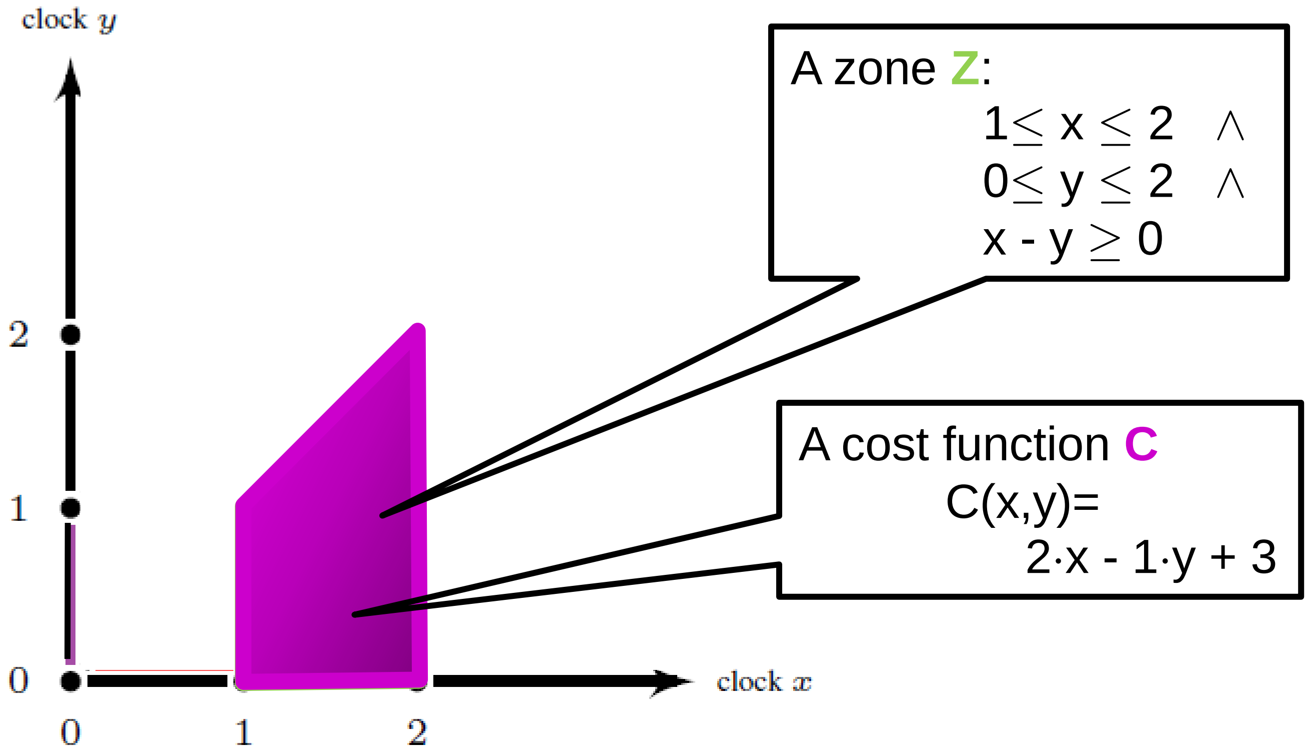
THM [Behrmann, Fehnker ..01] [Alur,Torre,Pappas 01]
Optimal reachability is decidable for PTA

THM [Bouyer, Brojaue, Briuere, Raskin 07]
Optimal reachability is PSPACE-complete
for PTA

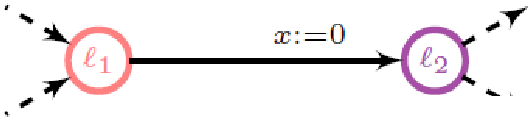


Priced Zones

[CAV01]



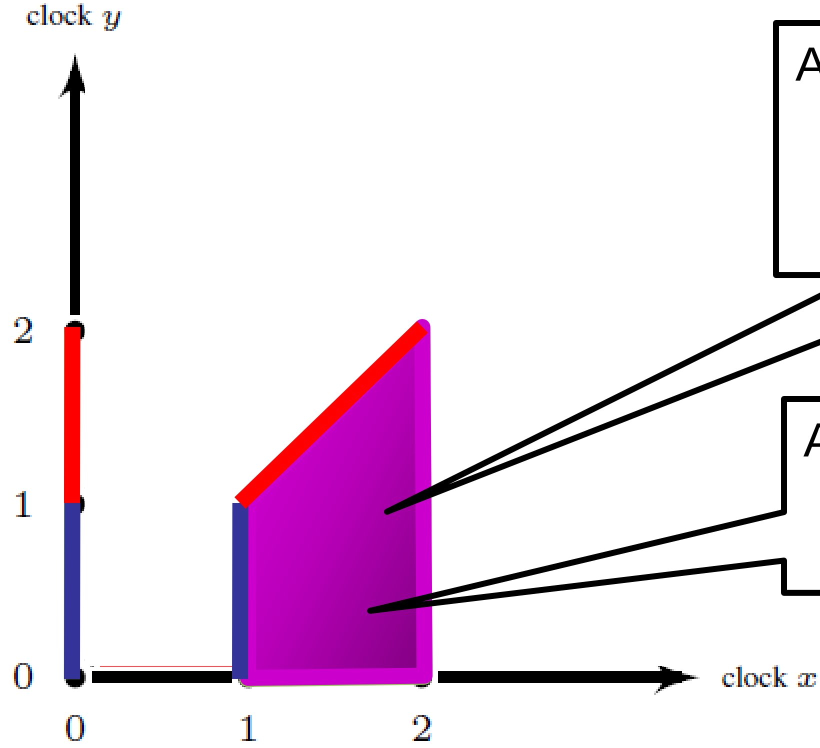
Priced Zones – Reset [CAV01]



Z[x=0]:
x=0 ∧
0≤ y ≤ 2

C = 1·y + 3

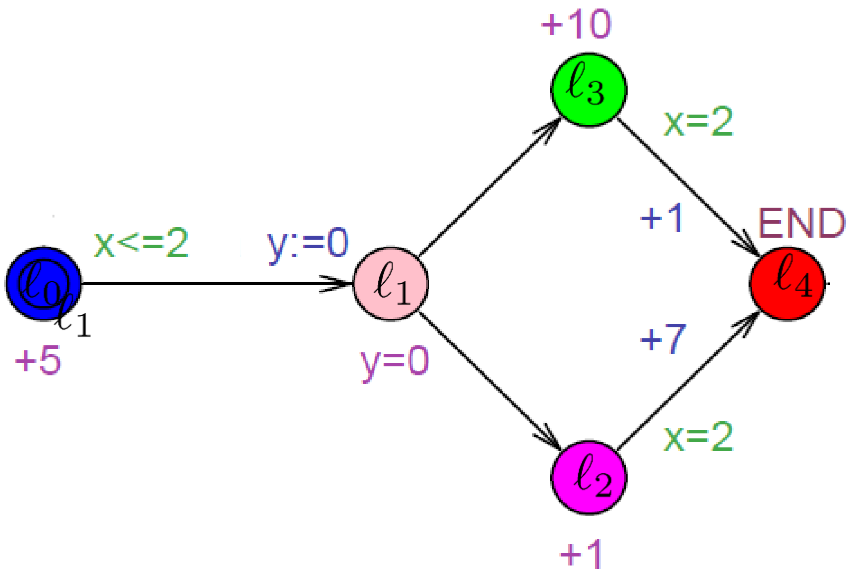
C= -1·y + 5



A zone **Z**:
1≤ x ≤ 2 ∧
0≤ y ≤ 2 ∧
x - y ≥ 0

A cost function **C**
C(x,y) =
2·x - 1·y + 3

Optimal Schedule



$$(\ell_0,[0,0]) \xrightarrow{1.2}_{6.0} (\ell_0,[1.2,1.2]) \rightarrow_0 (\ell_1,[1.2,0]) \rightarrow_0$$

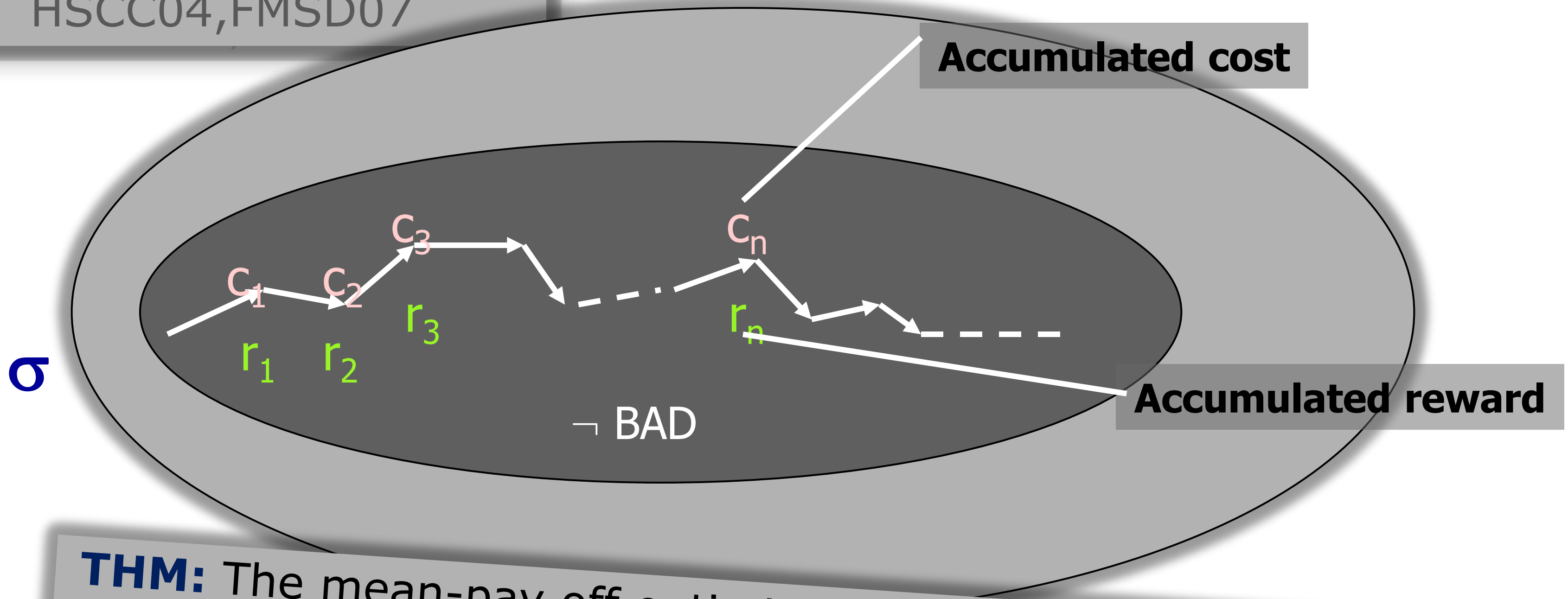
$$(\ell_3,[1.2,0]) \xrightarrow{0.8}_{8.0} (\ell_3,[2,0.8]) \rightarrow_1 (\ell_4,[2,0.8])$$

$$\rightarrow_{2.0} (\ell_0,[0,0])$$

$$\sum_i C_i / \sum_i t_i = 17/2 = 8.5$$

Mean Pay-Off Optimality

Bouyer, Brinksma, Larsen: HSCC04,FMSD07



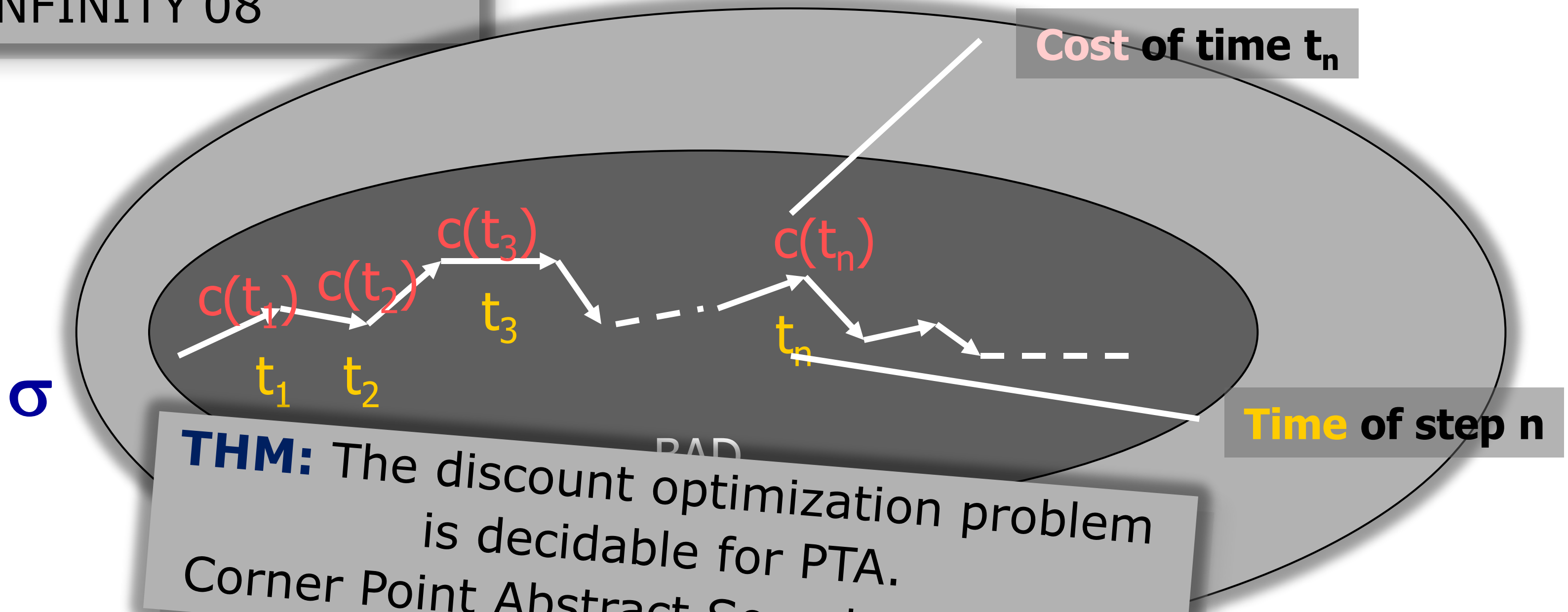
THM: The mean-pay off optimization problem is decidable (and PSPACE-complete) for PTA.
Corner Point Abstract Sound & Complete

Optimal Schedule σ^*: $val(\sigma^*) = \inf_\sigma val(\sigma)$

Discount Optimality

$\lambda < 1$: discounting factor

Larsen, Fahrenberg:
INFINITY'08

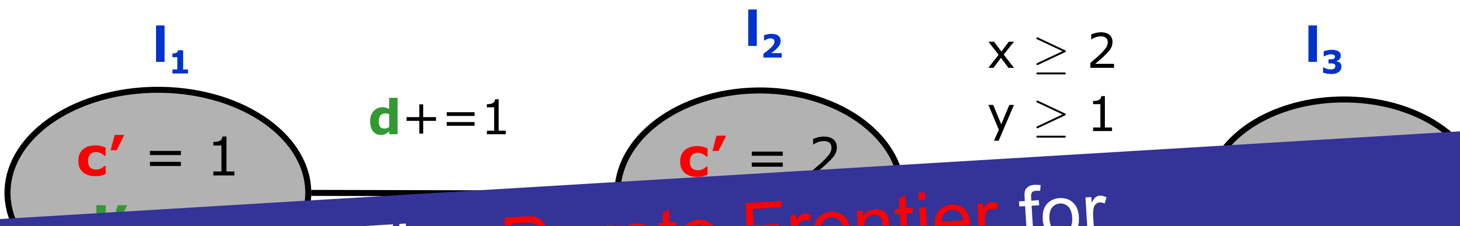


THM: The discount optimization problem is decidable for PTA.
Corner Point Abstract Sound & Complete

Value of path σ: $\mathrm{val}(\sigma) = \int_{t=0}^{t=\infty} c(t)\lambda^t dt$

Optimal Schedule σ^*: $\mathrm{val}(\sigma^*) = \inf_\sigma \mathrm{val}(\sigma)$

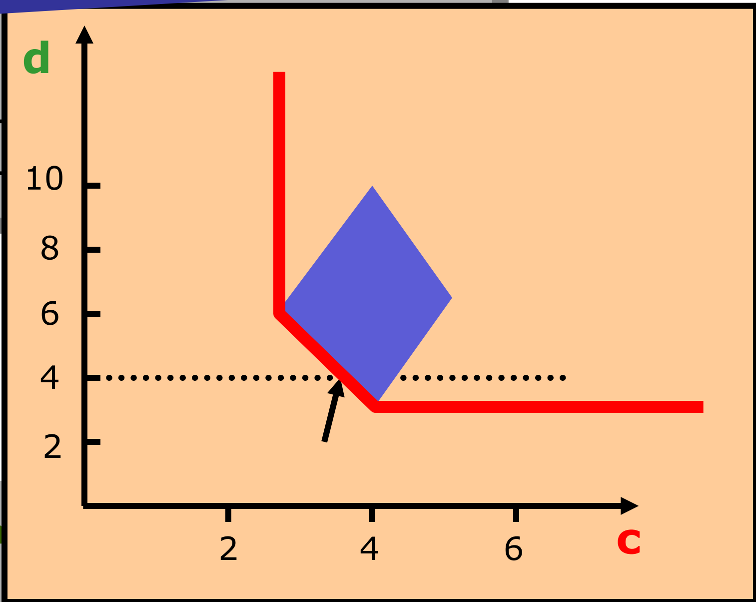
Multi Priced Timed Automata



The Pareto Frontier for
Reachability in Multi Priced Timed Automata
is computable
[Illum, Larsen FoSSaCS05]

Reach l_3 in a way which
minimizes **c**
subject to **d** ≤ 4

c =
wai
wai



Priced Timed Game

Decidable with **1** clock [BLMR06, HJM12]

- Acyclic [LTMM02]
- Bounded length [ABM04]
- Strong non-zeno cost-behaviour [BCFL04]

Undecidable with **3** clocks or more [BBR05, BBM06]

Open problem with **2** clocks

$$5t + 10(2 - t) + 1, 5t + (2 - t) + 7$$

P Bouyer, K G Larsen, N Markey, and J I Rasmussen. Almost optimal strategies in one clock priced timed games.. 2006
P Bouyer, Thomas Brihaye, and N Markey. Improved undecidability results on weighted timed automata.. 2006.
T Dueholm Hansen, R Ibsen-Jensen, and P Bro Miltersen. A faster algorithm for solving one-clock priced timed games. 2012.

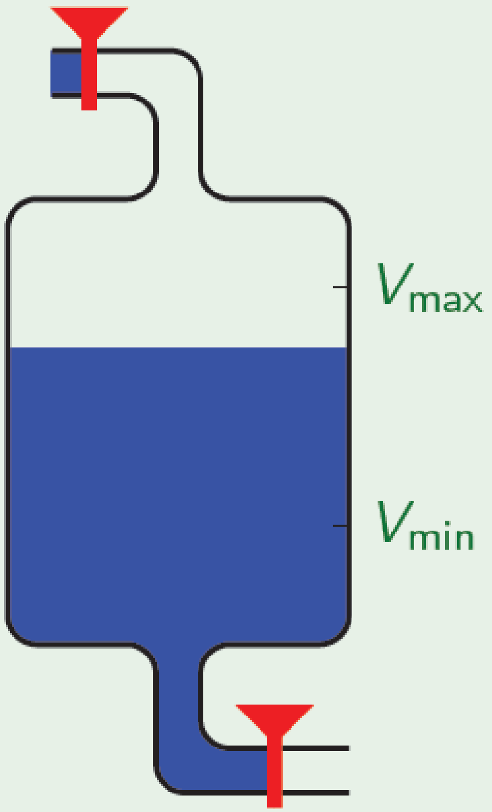
Energy Automata and Games

Consuming & Harvesting Energy

Example

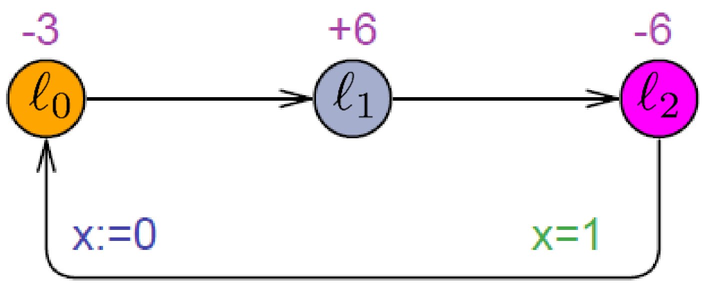
In some cases, resources can both be consumed and regained.

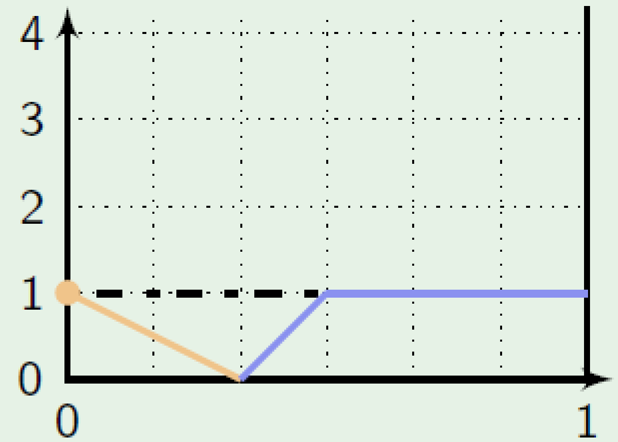
The aim is then to keep the level of resources within given bounds.



Energy Constrains

- Energy is not only consumed but may also be regained
- The aim is to continously satisfy some energy constriants





lower-weak-upper-bound problem

Results (so far)

Bouyer, Fahrenberg, Larsen, Markey, Srba: **FORMATS 2008**

Untimed

	games	existential problem	universal problem
L	$\in$ UP $\cap$ coUP P-h	$\in$ P	$\in$ P
L+W	$\in$ NP $\cap$ coNP P-h	$\in$ P	$\in$ P
L+U	EXPTIME-c	$\in$ PSPACE NP-h	$\in$ P

1 Clock

	games	existential prob	l problem
L	?	$\in$ P	$\in$
L+W	?	$\in$ P	$\in$ P
L+U	undecidable	?	?

Corner Point Abstraction Suffice

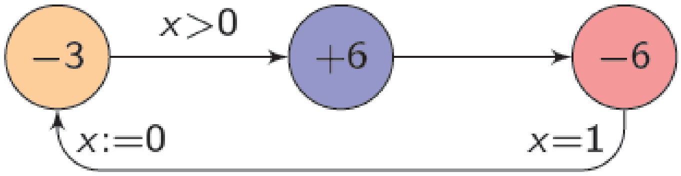
P Bouyer, U Fahrenberg, K Larsen, N Markey,.. . Infinite runs in weighted timed automata with energy constraints. 2008.

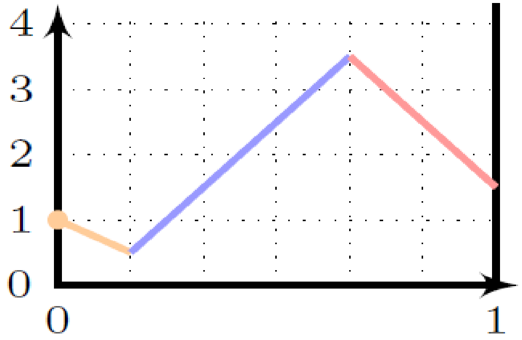
L–Problem for 1–Clock Case

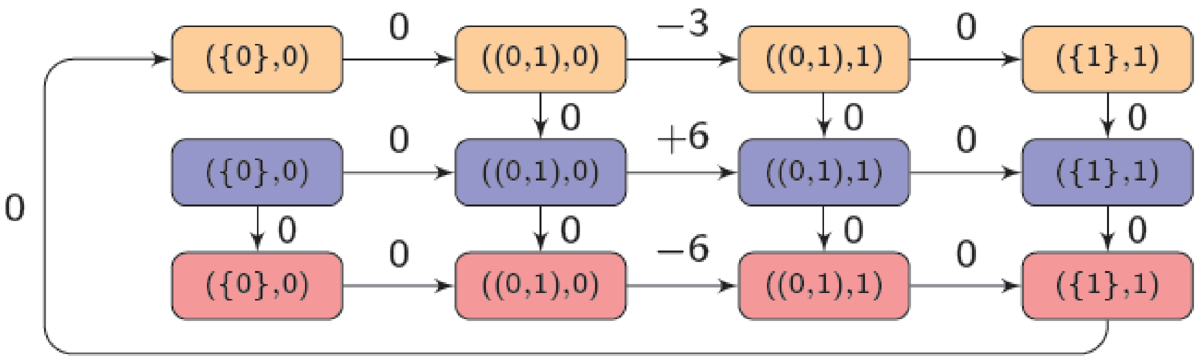
Theorem:
The L-problem is decidable in **PTIME** for **1**-clock PTAs

Proof.

- Corner-point abstraction:







P Bouyer, U Fahrenberg, K Larsen, N Markey,.. . Infinite runs in weighted timed automata with energy constraints. 2008.

LU–Problem for 1–Clock Energy Games

Theorem
For 1-clock priced timed games, the existence of a strategy satisfying LU-bounds is **undecidable**

Proof.

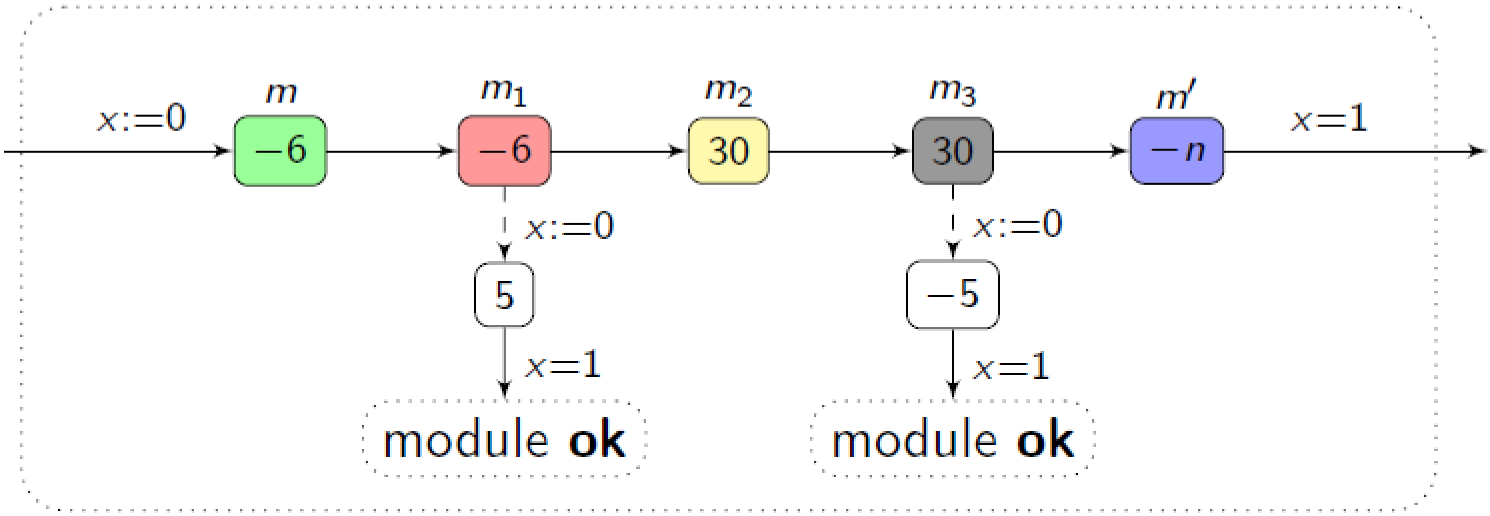
- we encode a 2-counter machine:
  - each instruction is encoded as a module;
  - the values c_1 and c_2 of the counters are encoded by energy level

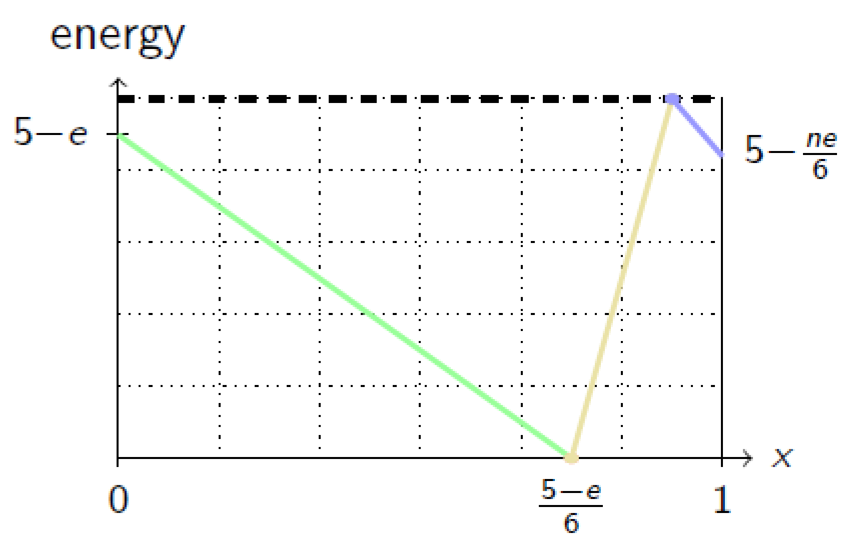
$$e = 5 - \frac{1}{2^{c_1} \cdot 3^{c_2}}$$

when entering the corresponding module.

P Bouyer, U Fahrenberg, K Larsen, N Markey,.. . Infinite runs in weighted timed automata with energy constraints. 2008.

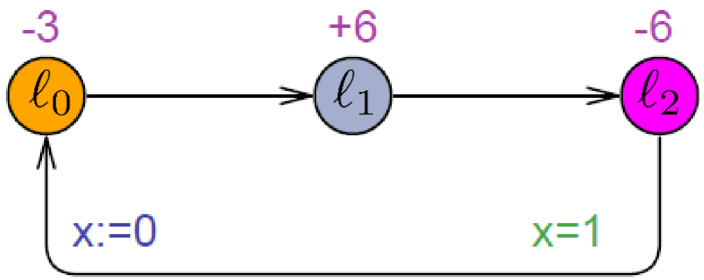
Generic Module for Inc/Dec

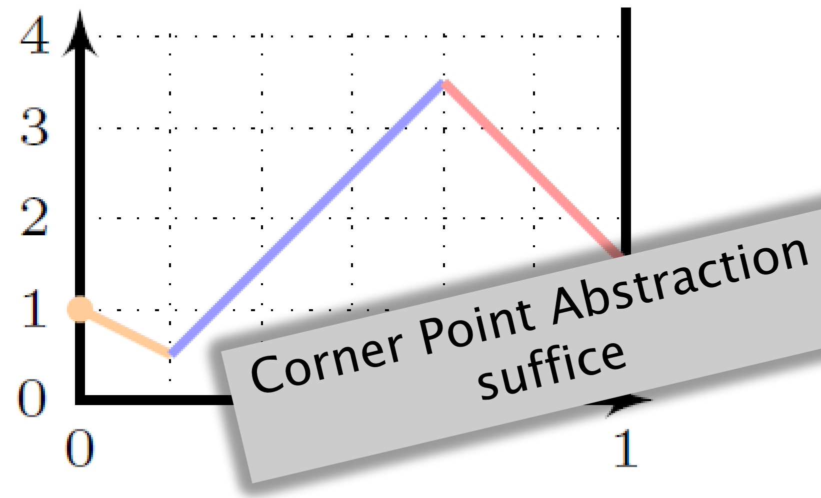


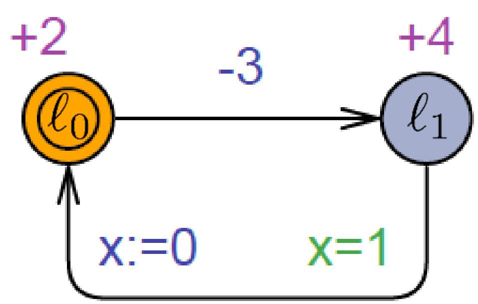


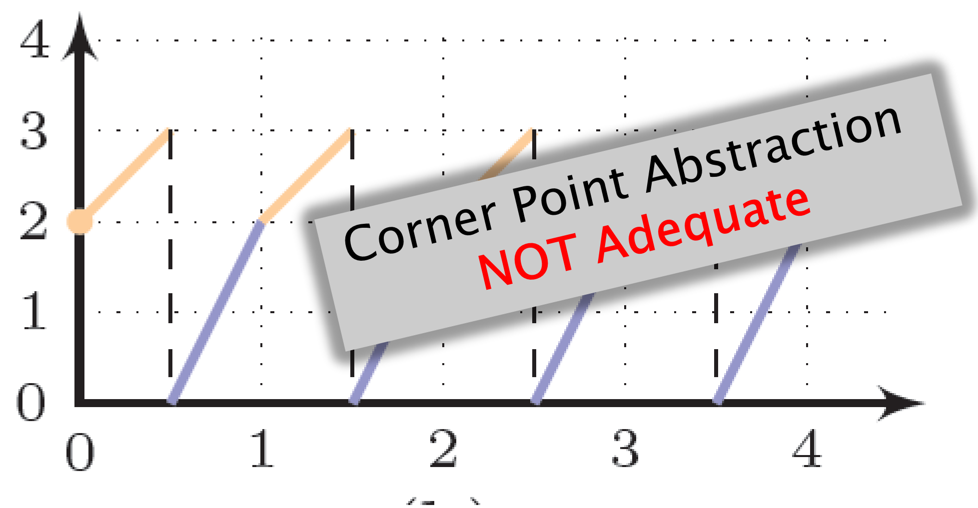
- $n = 3$: increment c_1
- $n = 2$: increment c_2
- $n = 12$: decrement c_1
- $n = 18$: decrement c_2

1½ Clocks = Discrete Updates

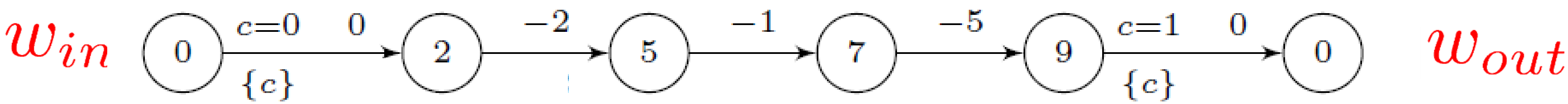




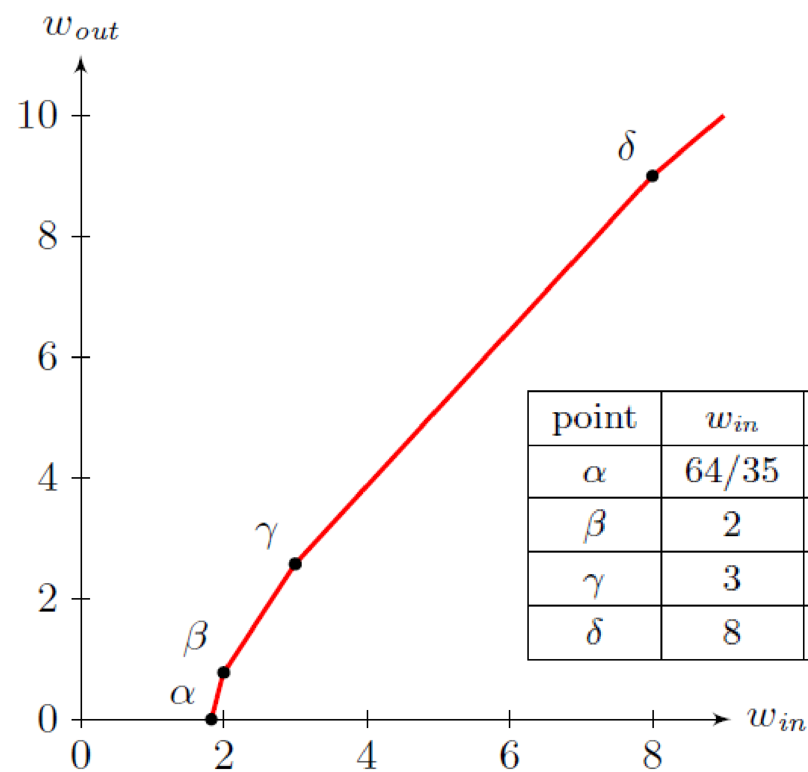




New Approach: Energy Functions



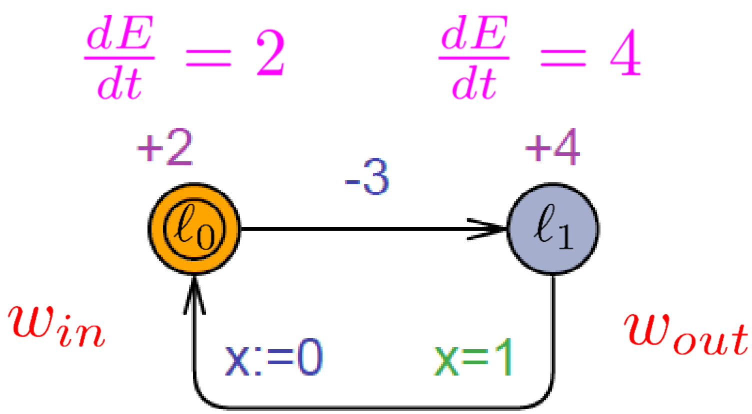
- Maximize energy along paths
- Use this information to solve general problem



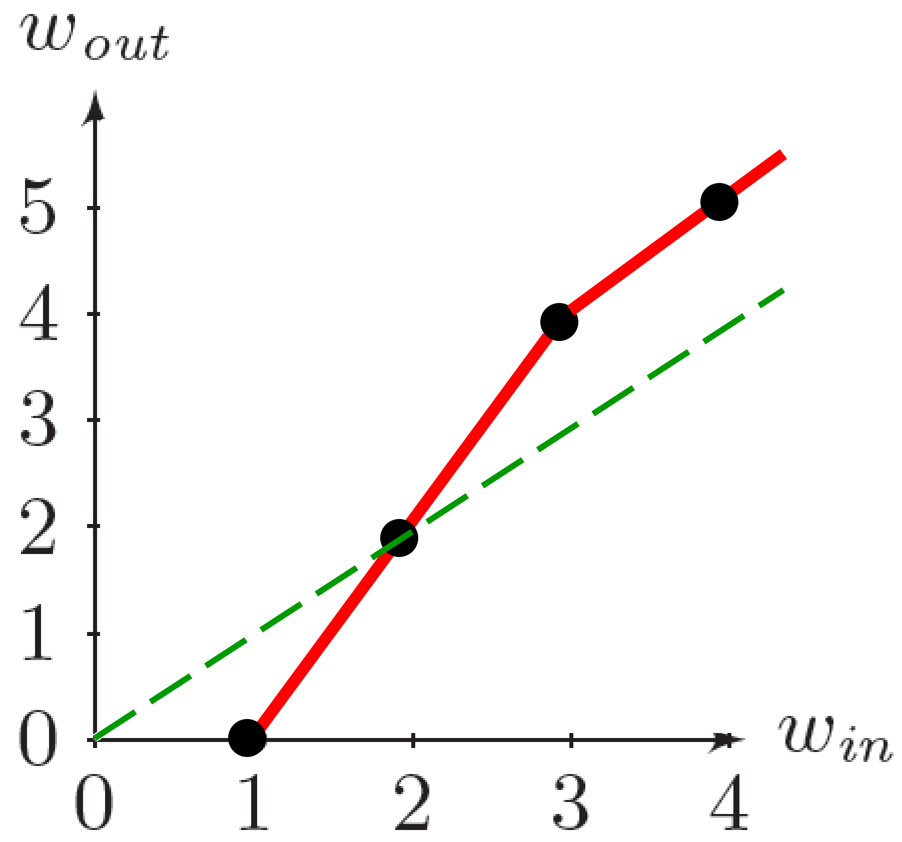
point	w_{in}	w_{out}
α	64/35	0
β	2	27/35
γ	3	18/7
δ	8	9

P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010

Energy Function

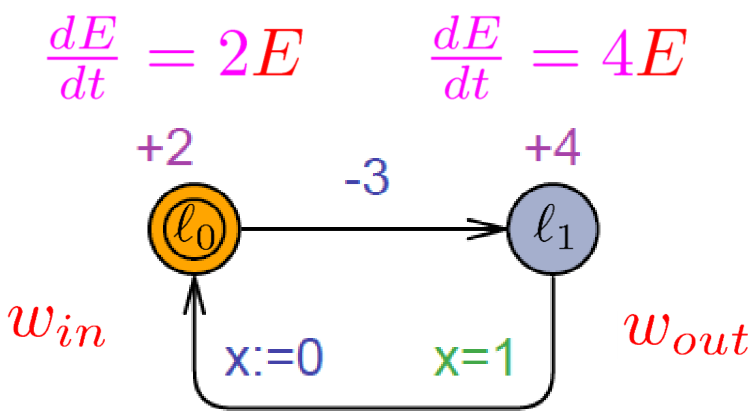


General Strategy
Spend just enough time to survive the next negative update

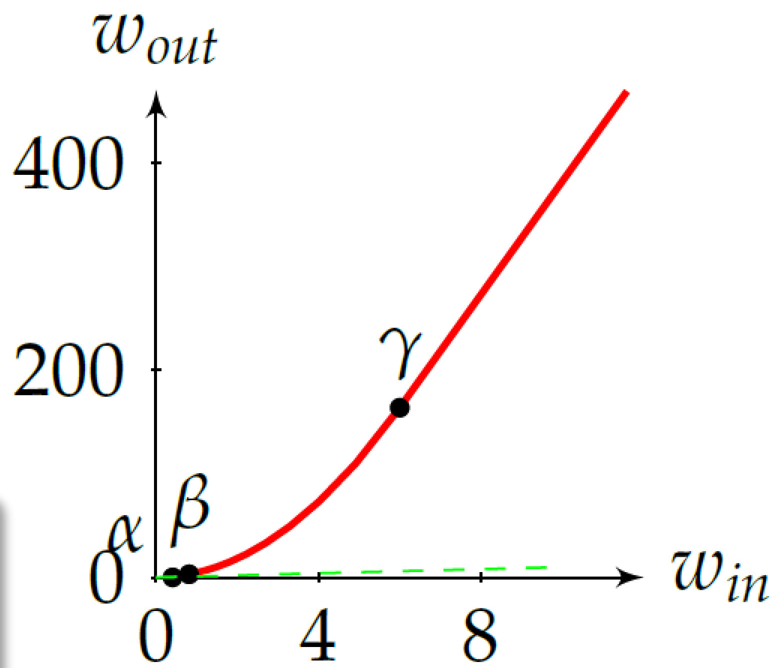


P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010

Exponential PTA



General Strategy
Spend just enough time
~~to survive the next negative update~~
so that after next negative update there is a certain positive amount !

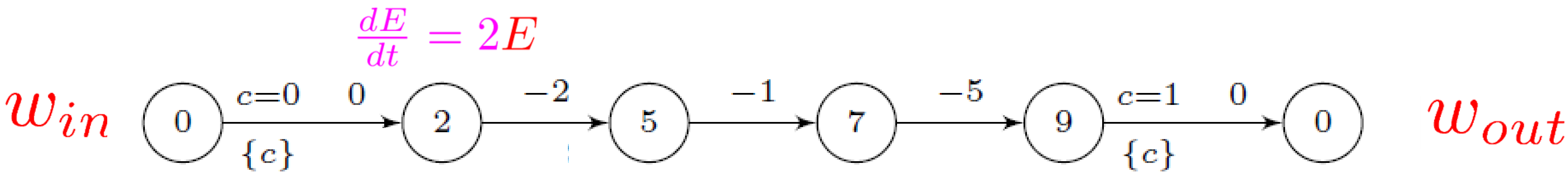


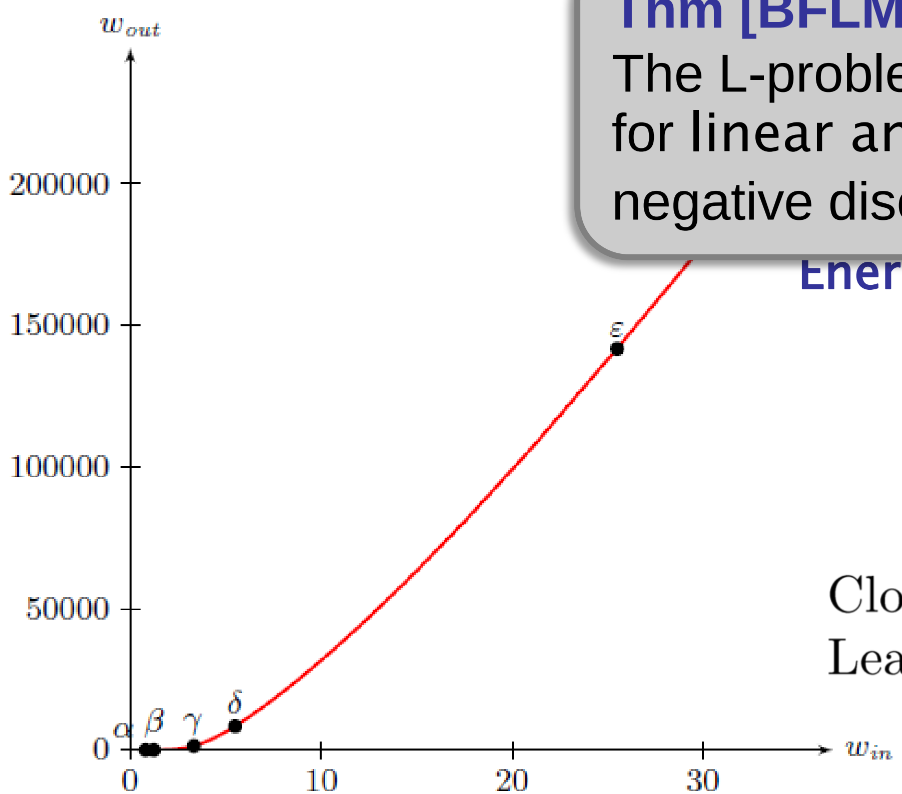
Minimal Fixpoint:

$$\frac{3}{e^2-1} \approx 0.47$$

P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010

Exponential PTA





Thm [BFLM09]:
The L-problem is **decidable**
for linear and exponential 1-clock PTAs with negative discrete updates.

Energy Function

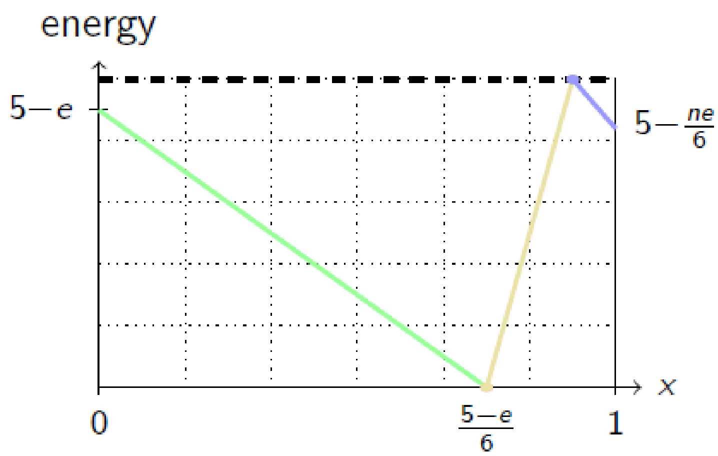
- $f : x \mapsto \alpha \cdot x^r + \beta$ where r is rational
- $\frac{df}{dt} \geq 1$

Closed under max and composition.
Least fixed point computable.

P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010

Multiple Costs & Clocks

- LU Problem **Undecidable**
  - 2 clocks + 2 costs [1]
  - 1 clock + 2 costs [2]
  - 2 clocks + 1 cost [3]

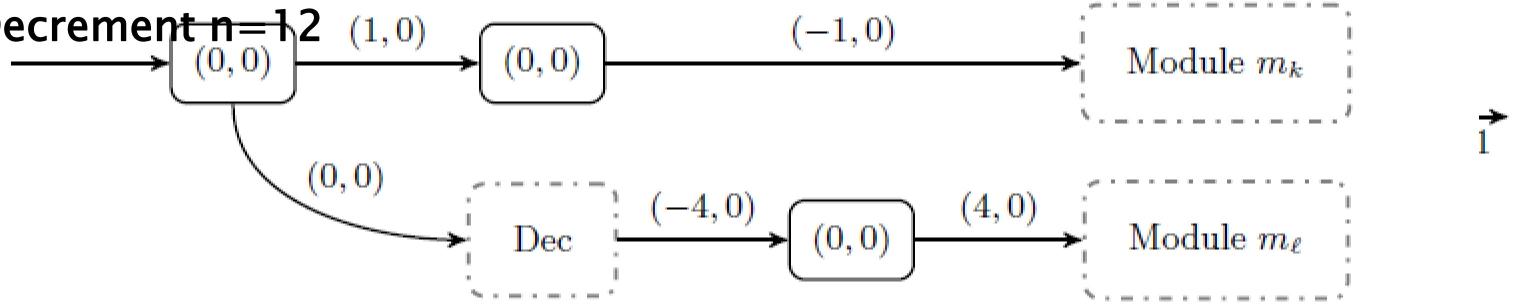


Update

Test-Decrement

Increment n=3

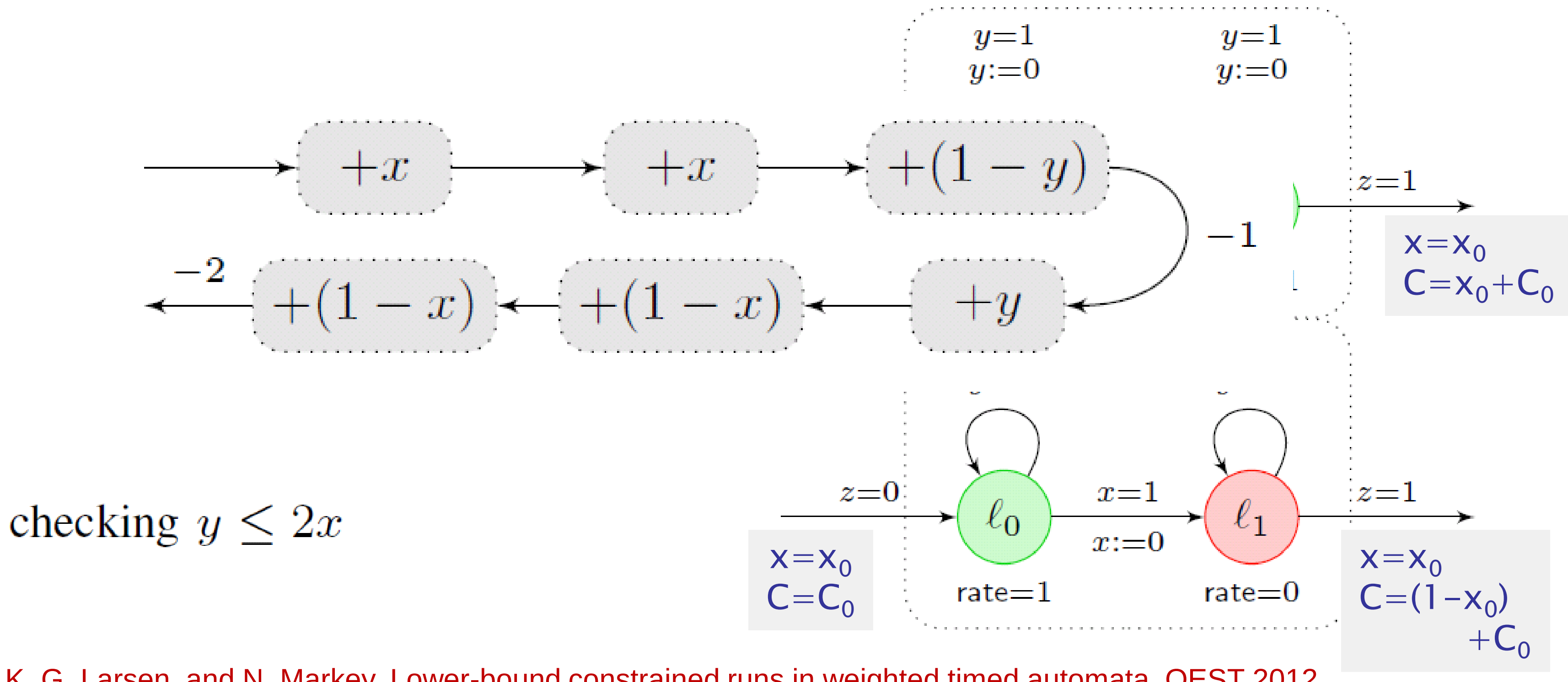
Decrement n=12



(1) Karin Quaas. On the interval-bound problem for weighted timed automata. 2011.
(2) Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jirı Srba. Energy games in multiweighted automata. 2011
(3) Nicolas Markey. Verification of Embedded Systems – Algorithms and Complexity. 2011.

Multiple Clocks & 1 Cost

- L problem is **undecidable**
  - 4 clocks or more



P. Bouyer, K. G. Larsen, and N. Markey. Lower-bound constrained runs in weighted timed automata. QEST 2012

Multiple Clocks & 1 Cost

L Problem

α \ Q	Fixed initial credit: $\exists$	Fixed initial credit: $\forall$	Existence of initial credit: $\exists$	Existence of initial credit: $\forall$
∞	≤ 1: decidable [8] ≥ 4: undecidable	in PSPACE ≥ 3: PSPACE-c.	in PSPACE ≥ 3: PSPACE-c.	
T	in NEXPTIME ≥ 5: NEXPTIME-c.			

Lemma 3. *If $\mathcal{A}$ has a feasible run ϱ from some (ℓ_0, v_0, c_0) to some (ℓ, v, c) of duration at most T, then it also contains a feasible run ϱ' of length[2] N in $O(T \cdot |X|^3 \cdot |L|^2)$, starting from (ℓ_0, v_0, c_0) and ending in (ℓ', v', c') such that*

- *either $(\ell', v') = (\ell, v)$ and $c' \geq c$ and $v'(u) = v(u)$,*
- *or from (ℓ', v', c') there is a profitable zero-delay cycle.*

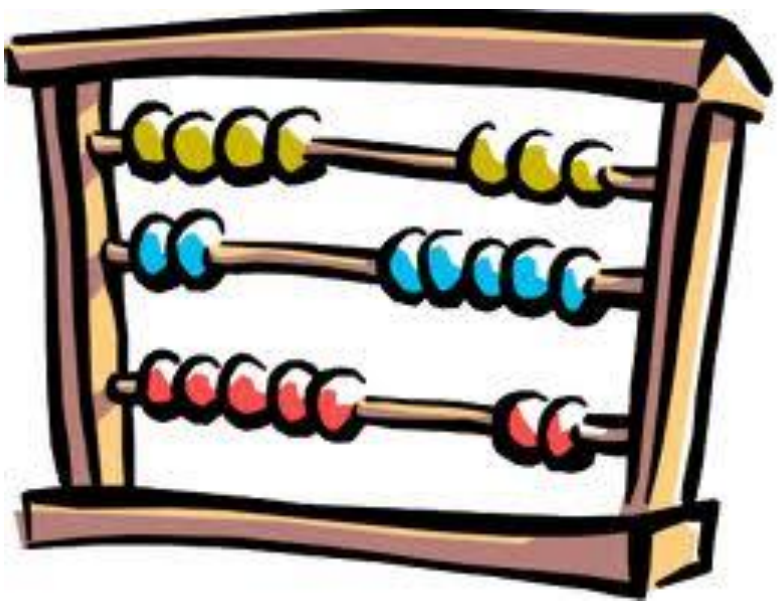
P. Bouyer, K. G. Larsen, and N. Markey. Lower-bound constrained runs in weighted timed automata. QEST 2012

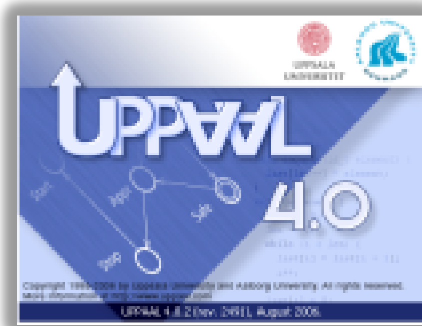
Multiple Costs & 0 Clocks

# weights	Bound	Existential	Universal	Game
One	L	∈ P [4]	∈ P [4]	∈ UP ∩ coUP [4]
	LW	∈ P [4]	∈ P [4]	∈ NP ∩ coNP [4]
	LU	NP-hard [4], ∈ PSPACE [4]	∈ P [4]	EXPTIME-complete [4]
Fixed ($k > 1$)	L	**NP-hard**, ∈ k-EXPTIME [3] (Remark 17)	**∈ P** (Remark 18)	**EXPTIME-hard**, ∈ k-EXPTIME [3] (Remark 19)
	LW	**NP-hard, ∈ PSPACE** **PSPACE-complete** for $k \geq 4$ (Remark 20)	**∈ P** (Remark 18)	**EXPTIME-complete** (Remark 21)
	LU	**PSPACE-complete** (Remark 20)	**∈ P** (Remark 18)	**EXPTIME-complete** (Remark 21)
Arbitrary	L	**EXPSPACE-complete** (Theorem 9)	**∈ P** (Remark 18)	**EXPSPACE-hard** (from EL) decidable [3]
	LW	**PSPACE-complete** (Theorem 9)	**∈ P** (Remark 18)	**EXPTIME-complete** (Remark 21)
	LU	**PSPACE-complete** (Theorem 9)	**∈ P** (Remark 18)	**EXPTIME-complete** (Remark 21)

Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jirı Srba. Energy games in multiweighted automata. 2011

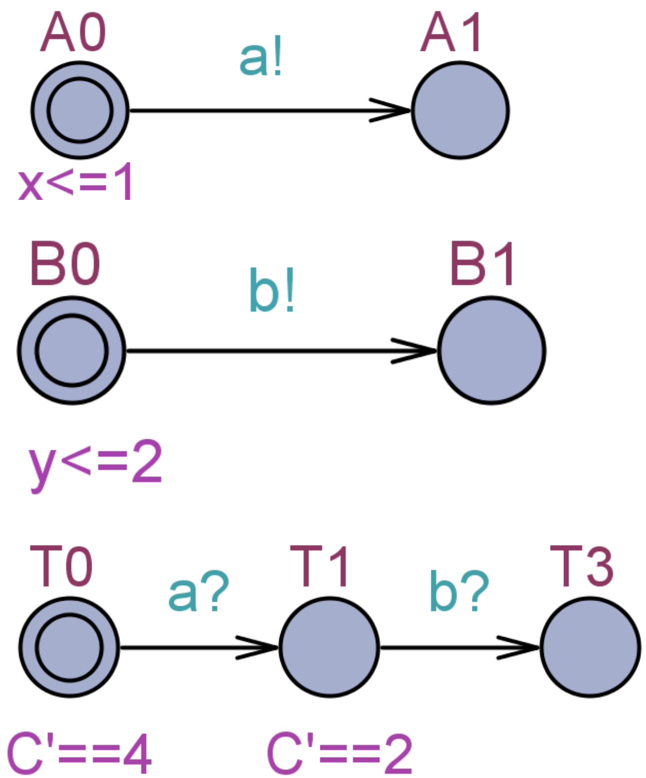
Practice

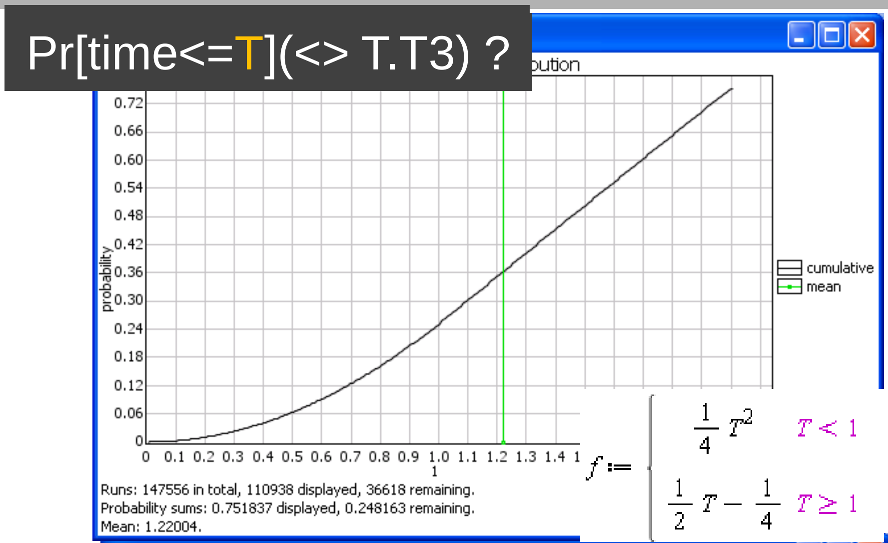


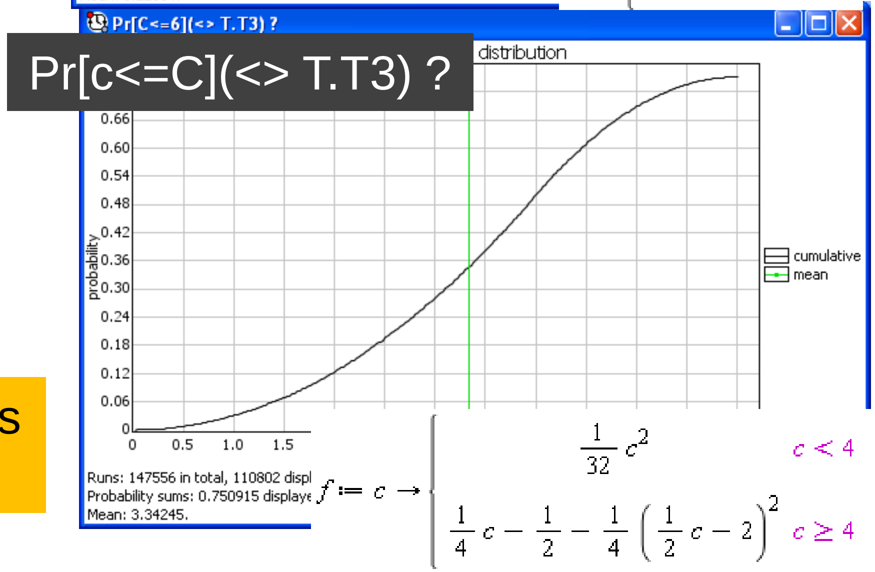


SMC

Stochastic Semantics of Timed Automata





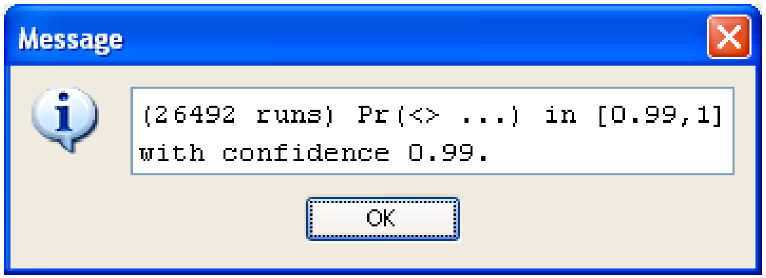


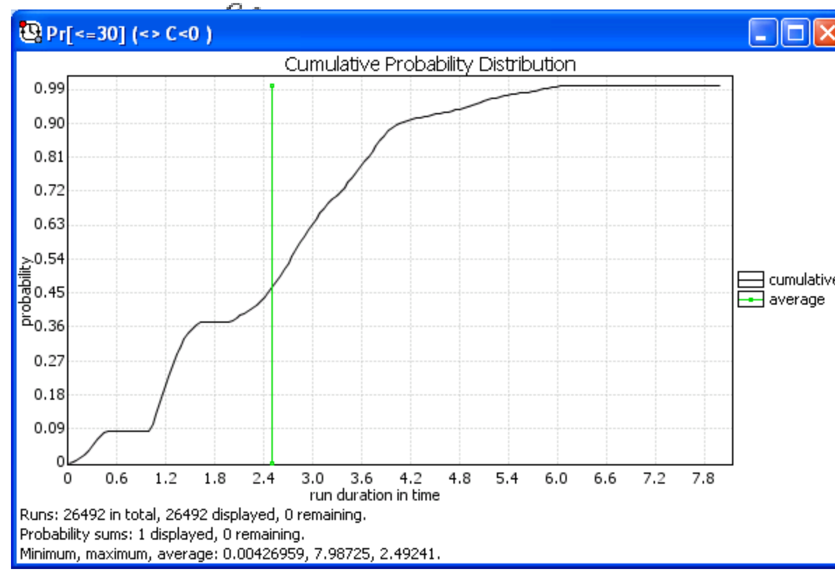
Composition = Race between components for outputting

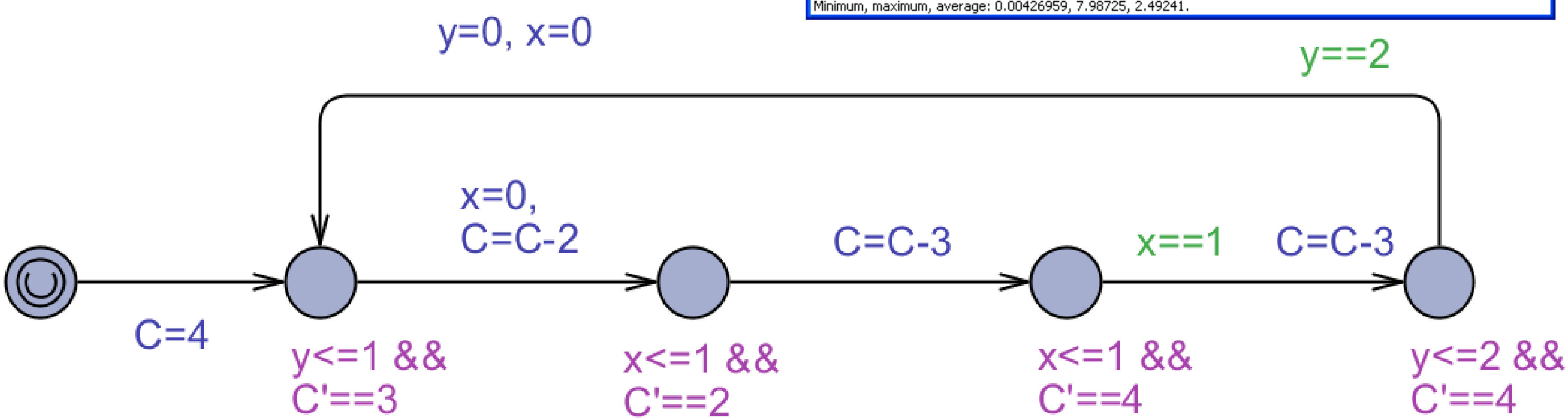
C. Baier, N. Bertrand, P. Bouyer, T. Brihaye, ..: Almost-Sure Model Checking of Infinite Paths in One-Clock Timed Automata. 2008.
A. David, K. G. Larsen, A. Legay, M. Mikucionis, .. : Statistical Model Checking for Networks of Priced Timed Automata. 2011.
A. David, K. G. Larsen, a. Legay, M. Mikucionis, Z. Wang: Time for Statistical Model Checking of Real-Time Systems. 2011.

Time Bounded L-problem

Pr[time<=30] (<> C<0)

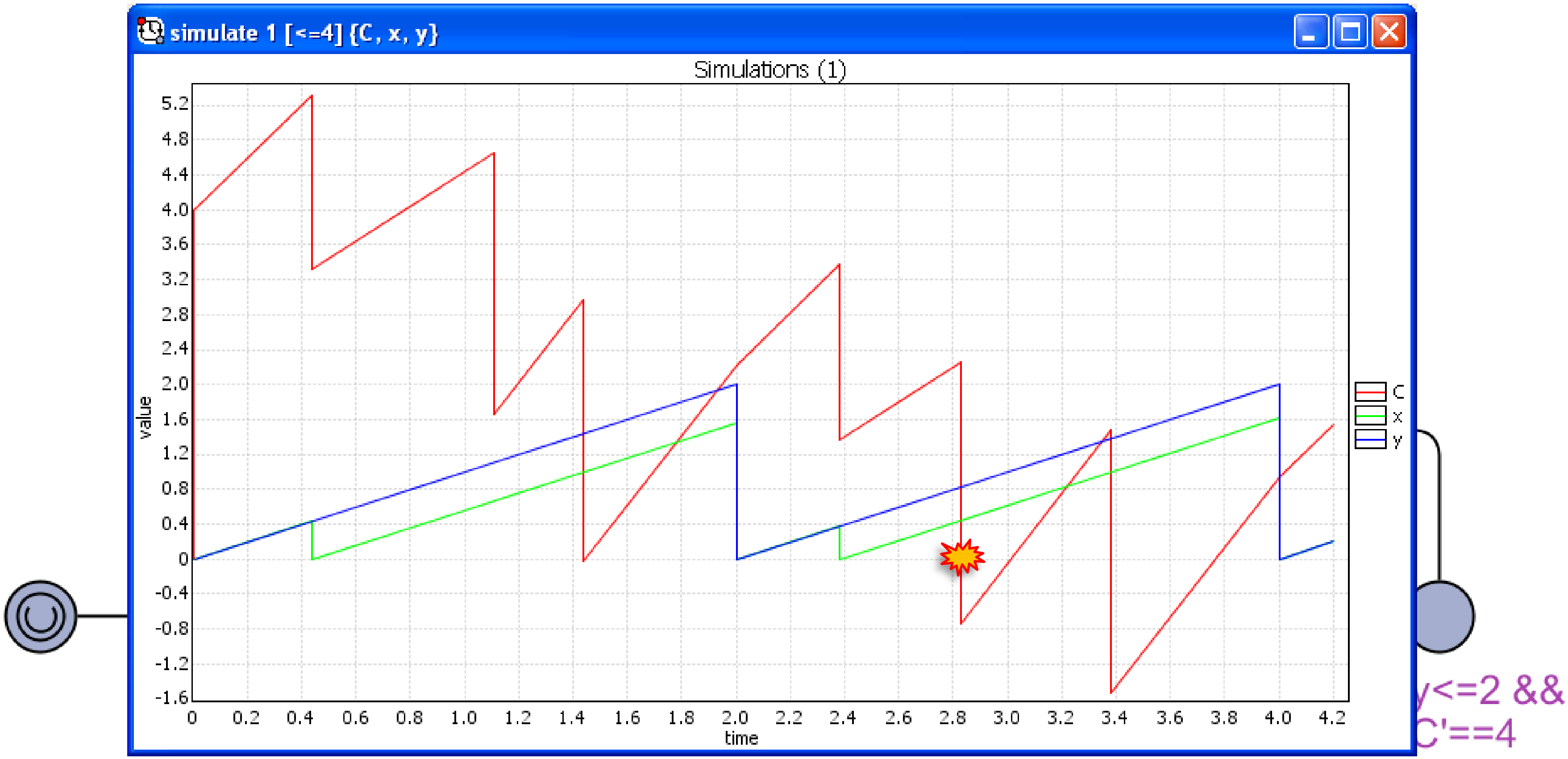






Time Bounded L-problem

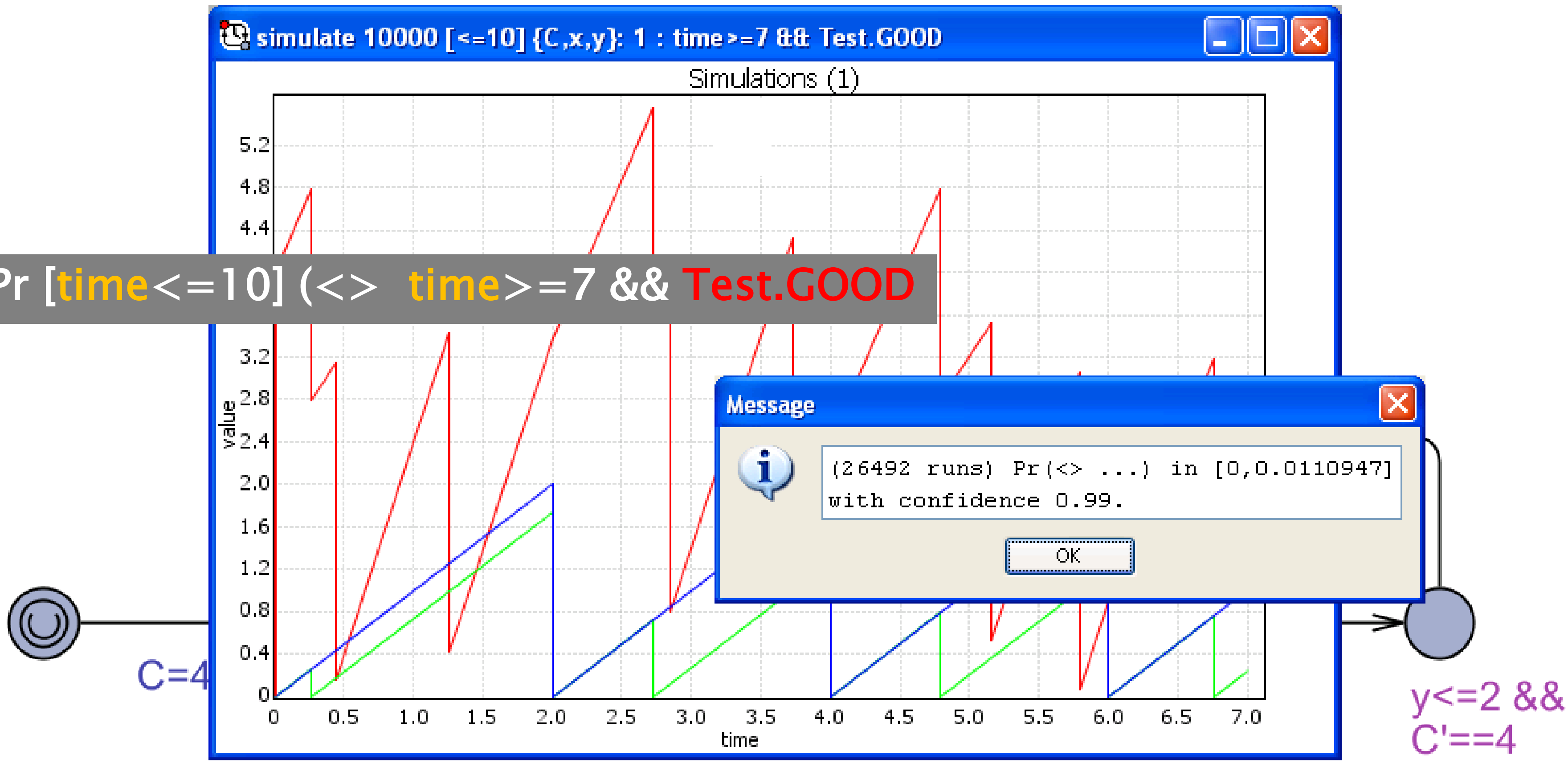
simulate 1 [time<=5] {C, x, y}



Time Bounded L-problem

simulate 10000 [time<=10] {C,x,y}: 1 : time>=7 && Test.GOOD

Pr [time<=10] (<> time>=7 && Test.GOOD



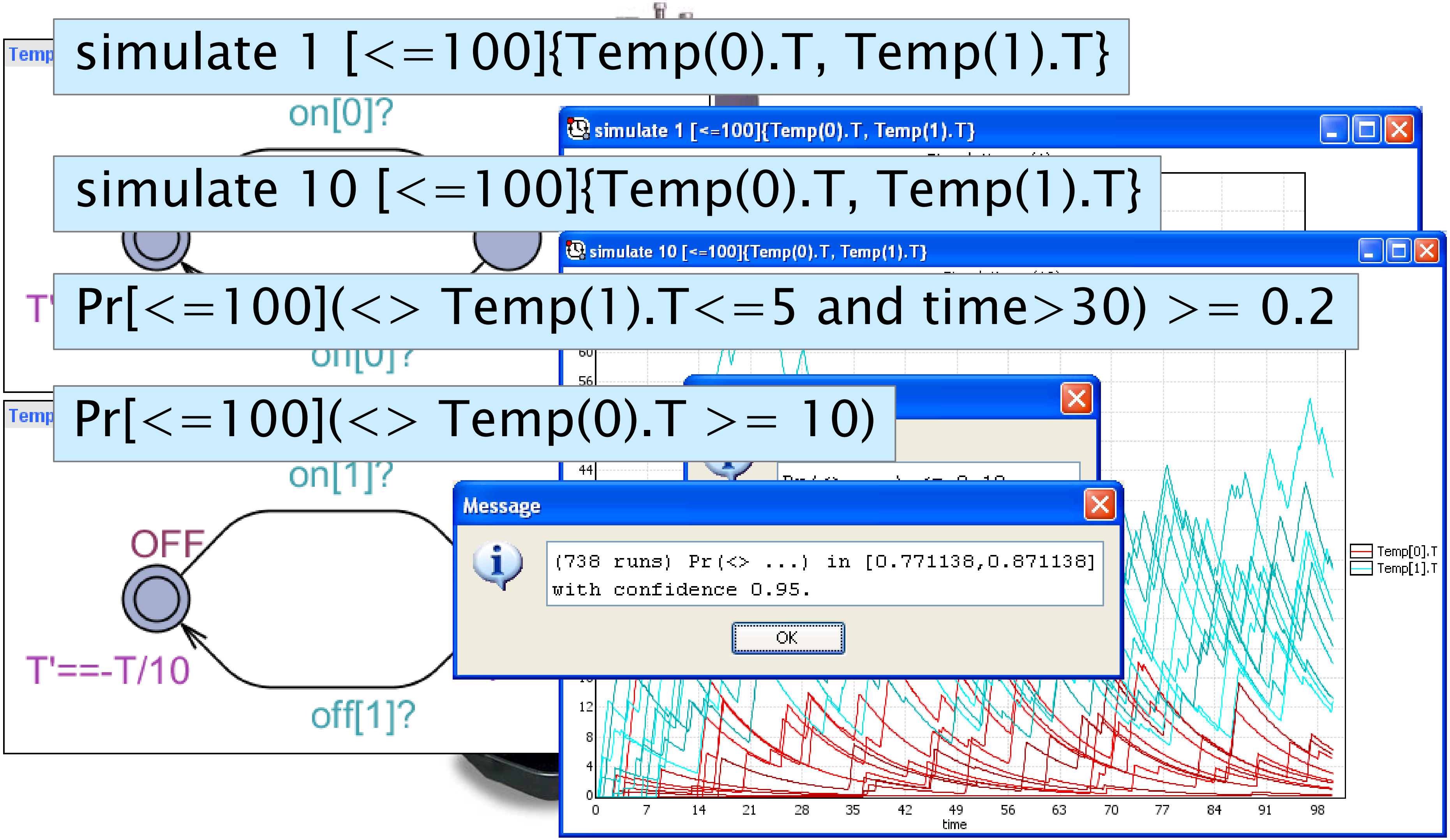
Stochastic Hybrid Systems

simulate 1 [<=100]{Temp(0).T, Temp(1).T}

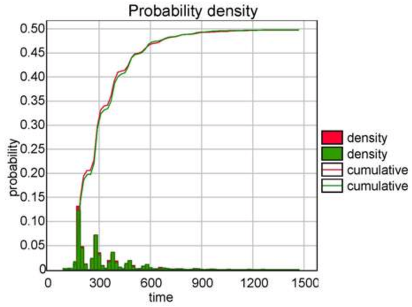
simulate 10 [<=100]{Temp(0).T, Temp(1).T}

Pr[<=100](<> Temp(1).T<=5 and time>30) >= 0.2

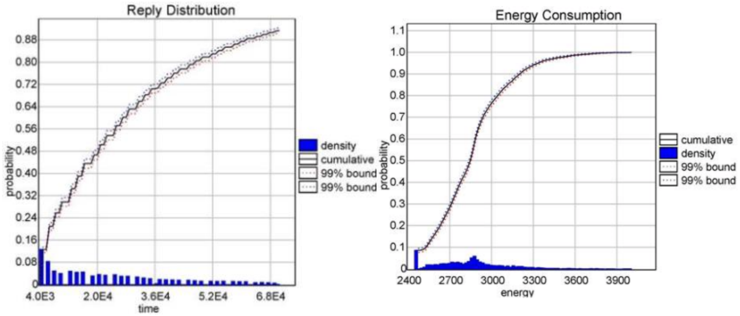
Pr[<=100](<> Temp(0).T >= 10)



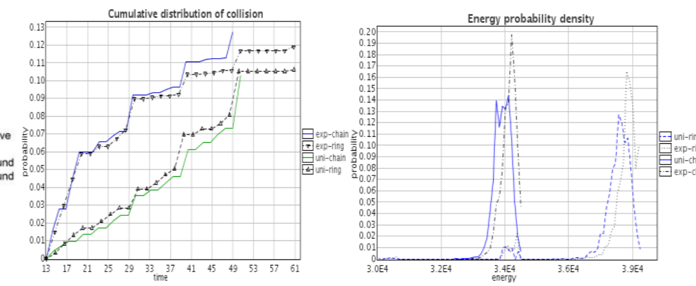
Case Studies



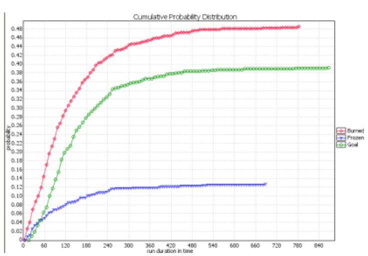
FIREWIRE



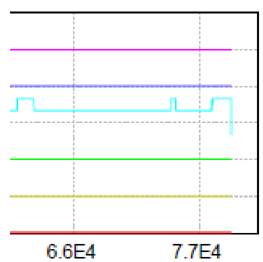
BLUETOOTH



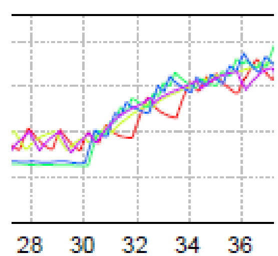
10 node LMAC



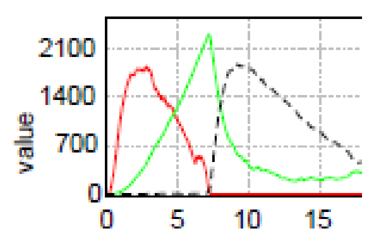
ROBOT



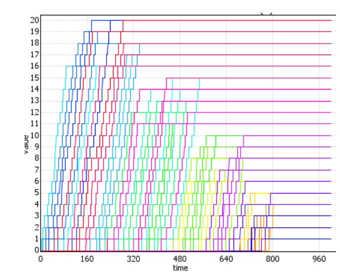
Schedulability Analysis for Mix Cr Sys



Energy Aware Buildings



Genetic Oscilator (HBS)



Passenger Seating in Aircraft

Conclusion

- Priced Timed Automata a uniform framework for modeling and solving dynamic ressource allocation problems!
- Future work:
  - Zone-based alg. for optimal infinite runs in PTA
  - Approximate solutions for PTG
  - Open LU-problems for EA
    - 0 clocks, 1 cost (NP-hard vs PSPACE)
    - 1 clock, 1 cost (decidability ?)
  - Application of Statistical Model Checking
  - SENSATION Project