Array programming languages

Alexander Harms

Institute for software engineering and programming languages

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Array programming languages

- Also known as vector or multidimensional languages
- Array oriented programming paradigms
- Multidimensional arrays as primary data structures
- Arrays can be vectors, matrices, tensors or scalar values
- Operations can apply to an entire set of data values without explicit loops of scalar operations
- Used for numerical simulations, data analysis and evaluation

What is the reason for array programming

- Often easier and faster than traditional programming languages
- Function libraries allows to avoid loops
- A fast and efficient way of solving numerical problems
- Fast computing by multicore processors combined with implizit parallelsization

- Can mostly be used without knowledge about computer architecture
- Very similar to common math notation
- Programmer can think about the data without thinking about how to handle loops

MATLAB

- MATLAB (Matrix LABoratory), developed by MathWorks
- Inital release in 1984
- Commercial product
- Multi-paradigm, numerical computing environment
- Main features: matrix manipulation, function and dataplotting, algorithm implementation and interfacing with other programs
- Fundamental data type is an n-dimensional array of double precision

- The MATLAB system consist of five parts
  - MATLAB as a language
  - working environment
  - graphic system
  - mathematical function library
  - application program interface

- MATLAB code is written in the command window, charaterized by the prompt '>>'

- Code is like interactive calculations or executing text files

Declaring variables

```
>> x= 3
x =
   3
>>x = ‘Hello‘;
>>array = 1:5:21
array =
   1 5 10 15 20
>> A = [1 2 3; 4 5 6; 7 8 9 ]
A =
   1 2 3
   4 5 6
   7 8 9
```

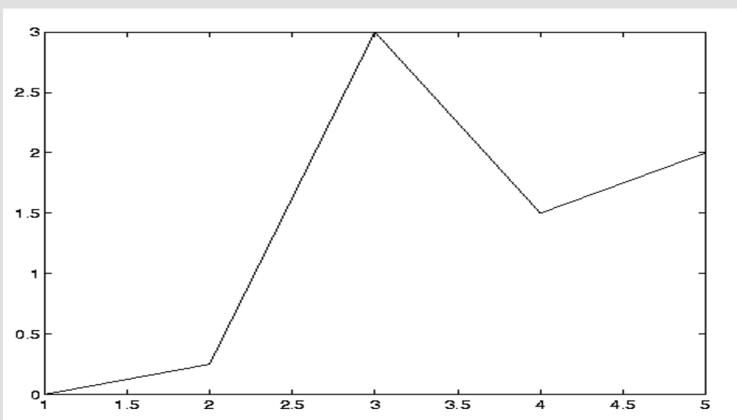
Matlab provides a powerful function library

```
>>array = linspace (0, pi , 7)
array =
  1   0.5236    1.0472    1.5708    2.0944    2.6180    3.1416
>>sum(array)
ans =
  11.9956
>> A = [1 1 1; 2 2 2; 3 3 3];
>> B = zeros(3:3);
>>A.*B
ans =
  1 1 1
  2 2 2
  3 3 3
```

- No loops needed, so developer can focus on numerical problem
- Off-by-one mistakes and others are ruled out
- Programs are saved as MATLAB-files (m-files)
- m-flies are scripts or functions
- Scripts are a collection of commands, functions run calculations
- MATLAB can also handle graphics

MATLAB graphics

```
>> x = [ 1; 2; 3; 4; 5];
>> y = [ 0; 0.25; 3; 1.5; 2];
>>figure      % opens new figure window
>>plot(x,y)
```



APL

- Named after a 1964 published book ‘A Programming Language‘

- Developed by Dr. Kenneth E. Iverson(1920-2004)
  - Mathematician
  - Special interest in mathematical notation
  - Disliked the standart notation
  - developed new system in 1957
  - Started to work for IBM in 1960
  - Wrote extended version (‘Iverson‘s Better Math‘)
  - IBM disliked the name, changed to ‘A Programming Language‘

- APL was a notation for expressing mathematical expressions first

APL‘s special set of non-ASCII symbols

- APL code is evaluated from right to left
- Just a few rules like parentheses for changing evaluation order
- Interativly written
- No need to declare variables
- Basic data structure is the array
- Special set of non-ASCII symbols is needed

APL code example

```
   3x4+5
27
   (3x4)+5
17
   x← 5
   x←“Hello“
A← 5 + 1 2 3            ⍝ A = 6 7 8
   B←  1 1 1
   A+B
7 8 9
   6 8 1 ⌊ 3 5 9
3 5 1
```

APL code can be very short. For the problem “Sum of 1 to 5“ a C-solution could be:

```
Include <stdio.h>
Int main(){
  int I = 0;
  int sum = 0;
  for (I = 1; i<=5; sum += i++);
  printf(“Sum: %d/n”, sum);
  return 0;
 }
```

The APL-solution would be:

```
+/ι 5
```

- Easy handling of array data
- Powerful function library, no need for counts or loops
- Code can be written very short
- Amount of unusual symbols leads to cryptic code
- Challenge to write APL ‘one-liner‘ is hard to resist

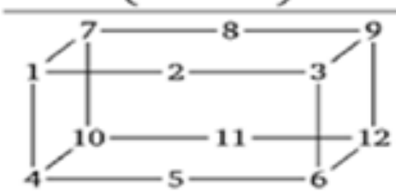
```
life←{↑1 ω∨.^3 4=+/,¯1 0 1∘.⊖¯1 0 1∘.⌽⊂ω}
```

- APL works best on multi core processors combined with implicit parallelization
- Mostly used in small projects, universities and research institutes
- Lateron, Iverson developed ‘J‘
- based on APL, but uses only ASCII-symbols

QUBE

- Developed by K. Trojahner in 2011
- Use dependent array types to check programs at compile time
- Combination of type checking and automatic theorem proving
- Rules out large classes of array boundary violations

- Arrays are characterized by their rank and shape
- The rank is a natural number of ist number of axes
- The shape describe the extent of each axis

Array	Rank	Shape vector
1	0	[]
[1 2 3]	1	[3]
$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$	2	[2 3]
	3	[2 2 3]

- QUBE extends $\text{QUBE}_{\text{core}}$ with a richer syntax, more base types and others like I/O

- $\text{QUBE}_{\text{core}}$ comprises three layers
  - QUBEλ (applied λ-calculus with dependent types)
  - QUBE→ (integer vectors)
  - QUBE[] (multidimensional arrays)

QUBE fun

- forms the basis of QUBE

- Most significant features
  - Dependend types
  - refinement types

- In following
  - x: values
  - T: types
  - e: expressions

Refinement type {x:T|e}

- Describes the subset of values x of txpe T that satisfy the boolean expression e.
- Example: the type ‘nat‘ are all integers with a value bigger than 0

$$type\ nat = \{x{:}int \mid 0 \leq x\}$$

- If e is true by any x, the type {x:T|true} is equivalent to T

Dependent funtion type x:T1→T2

- Binds the variable x of the domain type T1 in the codomain type T2
- Example: addition of two integer numbers

$$val+ : x:int \rightarrow y:int \rightarrow \{v:int \mid v=x+y\}$$

- Allows the result type to vary according to the supplied argument

Code example

```
let a = [1,2,3] in                (* vector constructor*)
let b = [1,2,1] in
let b = a.(2)<- a.(0) in          (* vector modification*)
a.(2)                             =>* a.(1) = 3
```

<u>QUBE vector</u>

- Adds support for integer vectors

- QUBE vector includes syntax for
  - defining,
  - accessing and
  - manipulating

  integer vectors

Vector constructor $[e]$

- Defines a vector with elements e.
- If all elements evaluate to integers, the resulting vector is a value

Vector selection e.(e_i)

- Selects the element at index e_i from the vector e

<u>Vector modification e.(ei) ← ee</u>

- The element at the index ei is replaced with the new element ee

<u>Constant-value vector expression vec en ee</u>

- defines a vector of non-constant length en that only contains copies of the element ee
- If the integer n is not negativ and the element evaluates to a value v, the entire expression evaluates to a vector that contains n copies of v.

```
let n = 1 + 3 in
vec n 0                 => *[0,0,0,0]
```

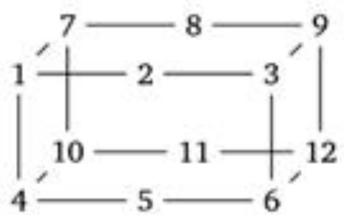
QUBE array

- Adds support for multidimensional arrays and rank-generic programing

Array constructor $[e : T|[n]]$

- defines a multidimensional array with elements e of the element type T and shape n (and thus rank $|n|$)
- The shape must be a natural number and the array elements must be equivalent to the product of the shape vector

QUBE arrays array constructor [e :T|[n]]

Array	Uniform array representation
1	[1 : int | []]
$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$	[1,2,3 : int | [3]]
$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$	[1,2,3,4,5,6 : int | [2,3]]
	[1,2,3,4,5,6,7,8,9,10,11,12 : int | [2,2,3]]

The example shows the array modification ea.[e] $\leftarrow$ ee

```
[1,2,3,4 :int |[2,2]].[[0,1]] <- 0          =>[1,0,3,4:int|[2,2]]
```

Conclusion

- Powerful for solving numerical problems and big matrices operations
- Large sets of functions, which allow to avoid error-prone loops
- APL programs are typically very short programs
- MATLAB provides a large set of external libraries for every situatuion
- QUBE focuses on ruling out as many mistakes at compile time as possible

Conclusion

- Array programming languages do not fit for every problem
- APLs code is very cryptic
- Fitting and user friendly graphic interfaces are rare
- Array programming is very powerful combined with multicore processors and implizit parallelization

Any questions?